# Chapter 12

# On the use of Modal Test Data in Inverse Problems: Fundamentals and Applications


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# On the use of Modal Test Data in Inverse Problems: Fundamentals and Applications

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**Abstract**

*In the past few decades, modal testing has become a major technology for determining, improving and optimizing dynamic characteristics of engineering structures in different fields, with emphasys (but not limited to) mechanical, civil, and aeronautical/aerospace engineering. One of the great advantages of using modal tests is related to their experimental practicality. This practical advantage leads to its strong use in inverse methods, or commonly known as inverse modal-based problems. An inverse problem based on modal parameters can be formulated basically using i) natural frequency, ii) damping factor or loss factor and iii) mode shapes. Other metrics can also be used from the composition of these 3 main responses, such as modal strain energy, inverse FRF and many others. Inverse modal-based identification through optimization algorithms are particularly emphasized. The methods discussed here are mainly elaborated by the evaluation of modal data due to the great potential of application. This chapter discusses the use of computational and intelligent techniques for parameter identification using modal responses. The content in this paper aims to help engineers and researchers find a starting point in developing a better solution to their specific structural problems, either by inverse methods, pattern recognition, or intelligent signal processing.*

**Keywords:** Modal Testing; Experimental Modal Analysis; Parameter Estimation; Inverse Problems.

## 1 Introduction and Context

The field of modal testing is quite extensive and, to be mastered, it is necessary to integrate knowledge from different engineering fields: vibration measurements, signal processing and structural analysis, including the necessary mathematical background. This Chapter does not intent to address in detail all aspects of experimental modal analysis, but only to familiarize and give the most important concepts and recent application in inverse problems to the reader. For a more comprehensive understanding of the topics covered, the novice researcher may rely on the vast classical books on the field, such as the theoretical foudations of vibration analysis found in Meirovitch [2010] and Inman [2013] or the fundamentals of modal testing, such as Silva and Maia [1999], Ewins [2000], Fu and He [2001] and Avitabile [2017].

According to Aster et al. [2012], the formulation of direct and inverse problems may be seen as an attempt to better understand physical phenomena in general by means of mathematical models, assuming that the underlying physical concepts are adequately understood. The idea is to relate physical parameters characterizing a model $\mathbf{m}$ to a set of data $\mathbf{d}$ by means of an operator $G$, such that

$$G(\mathbf{m}) = \mathbf{d}. \tag{1}$$

The model $\mathbf{m}$ may be understood as a set of physical parameters and relations; the data $\mathbf{d}$ may be a function of time and/or space, or may be a collection of discrete observations. The operator G can take on different forms, depending on the chosen approach. It can be an ordinary differential equation (ODE), or partial differential equation (PDE), or systems of ODE/PDE. In other cases, $G$ may be a linear or nonlinear system of algebraic equations. This terminology is not a consensus between mathematicians and other scientists. The former usually refer to $G(\mathbf{m}) = \mathbf{d}$ as the model and $\mathbf{m}$ as the set of parameters, while the latter call $G$ the forward operator and $\mathbf{m}$ the model. Without loss of generality, the second terminology is adopted hereafter.

The *forward* problem, or *direct* problem, is the process of obtaining $\mathbf{d}$ given $\mathbf{m}$, which usually involves solving ODEs or PDEs, solving a system of linear or nonlinear algebraic equations, evaluating integrals or even applying numerical algorithms when an explicit form of the operator $G$ is not available. On the other hand, the *inverse* problem is to find $\mathbf{m}$ given $\mathbf{d}$, which can be, in most cases, a much more complex endeavor than solving the associated direct problem. Solution existence, solution uniqueness ant instability of the solution are some of the issues that arise when dealing with inverse problems and that must be carefully addressed. For further information in the topic of inverse problems in strtuctural integrity problems, ranging from basic theoretical background to advanced applications, the reader is encouraged to consult the first volume of the present Book Series (Jorge et al. [2022]).

In this context, modal testing may be seen as an inverse problem in itself, when one focus on the process of identifying a useful dynamic model from measured data. The identified model may be a set of estimated Frequency Response Functions (FRFs) or, more frequently, a set of modal parameters, such as natural frequencies, damping factors and natural mode shapes, which can be used in adequate sets of ODE, PDE or algebraic equations in order to describe the dynamic behavior of the modeled system. Once obtained, the described modal data can, in turn, be used in several applications as *models* in direct/forward problems or as *data* in inverse problems.

The relevance and broad range of the use of modal data in engineering investigations in general, and in structural integrity problems in particular, is the main motivation for the present chapter. The fundamental concepts that are necessary for a basic understanding of the topic are briefly reviewed. Additionally, a selection of recent applications of the use of modal data in structural integrity problems is presented and discussed. The authors hope that the present chapter may provide the reader with the basic tools and information to further studies in the field, taking advantage of a comprehensive list of classical and recent literature for each relevant topic.

## 2 Overview and Philosophy of Modal Testing

Since the late 1970s, according to Fu and He [2001], modal analysis has increasingly become a major technology in the quest for determining, improving and optimizing dynamic characteristics of engineering structures. A very interesting historical overview of the first 30 years of developments in the field may be found in Brown and Allemang [2007]. Not only has the topic *modal testing* been recognized in mechanical and aerospace engineering, but modal analysis has also encountered profound applications for civil and building structures, bio-mechanical problems, acoustical instruments, transportation and nuclear plants, among others.

In the same way, according to Ewins [2000], since the very early days of awareness of structural vibration, experimental observations have been necessary for the major objectives of *i*) determining the nature and extent of vibration response levels in operation and *ii*) verifying theoretical models and predictions of the various dynamic (vibration) phenomena. There is also a third requirement, which is, *iii*) identifying the essential material properties under dynamic loading, such as damping capacity, friction and fatigue endurance. One of the major requirements of the subject of modal testing is a thorough integration of three components:

- The theoretical basis of vibration;
- Accurate measurement of vibration;
- Realistic and detailed data analysis.

Equally important, in summary, modal analysis is the process of determining the inherent dynamic characteristics of a system in forms of natural frequencies, damping factors and mode shapes, and using them to formulate a mathematical model for its dynamic behavior. The formulated mathematical model is referred to as the modal model of the system and the information for the characteristics are known as its modal data (Fu and He [2001]). Modal analysis embraces both theoretical and experimental techniques. The theoretical modal analysis anchors on a "spatial model" of a dynamic system comprising its mass, stiffness and damping properties. These properties may be given in forms of partial differential equations. Modern finite element (FE) analysis empowers the discretization of almost any structural system and hence has greatly enhanced the capacity and scope of theoretical modal analysis. On the other hand, the rapid development over the last two decades of data acquisition and processing capabilities has given rise to major advances in the experimental realm of the analysis, which has become known as modal testing.

Figure 1 depicts the theoretical and experimental vibration routes previously discussed (Covioli [2021]). On the theoretical path, the relevant geometrical and material properties of the system are the main input for the creation of the so-called "spatial model", most frequently a FE numerical discretization of a complex structure that combines several different continuous elements. This numerical representation in physical coordinates can be transformed through the solution of an algebraic eigenproblem into a model consisting of the natural frequencies and mode shapes of the system, which is referred to as the "modal model". From the modal model it is possible to efficiently obtain response functions in the time domain or in the frequency domain, allowing for a wide range of simulations of the dynamic behavior of the system when submitted to the envelope of excitations that will occur during its operation.

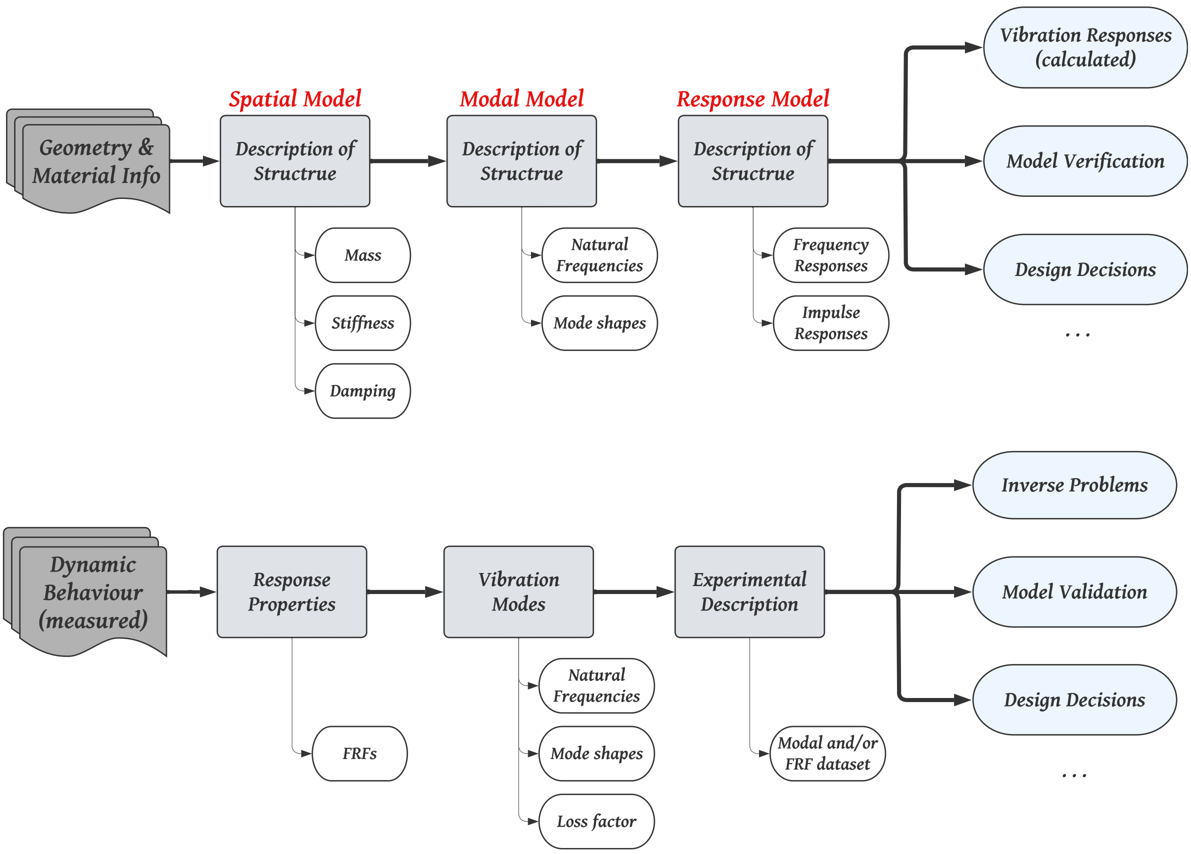



**Figure 1: Vibration analysis route : (a) theoretical ; (b) experimental (adapted from Covioli [2021]).**

On the experimental path, the starting point is the measurement of the dynamic behavior of the structure when conveniently excited. Signal acquisition and processing of the excitation forces[1] and vibration responses (most frequently accelerations or velocities of selected points of the structure) are the input data for the identification steps that follow.

[1]It is possible to perform modal tests without the measurement of the excitation forces, relying solely on the measurement of vibration responses. This is particularly useful when investigation systems in its

Response functions are then obtained and used to feed modal parameter extraction algorithms, resulting in an experimental modal model representing the vibration modes of the structure. Now, the dynamic behavior of the structure is described by a set of identified natural frequencies, modal damping factors and natural mode shapes. The experimental modal parameters, together with a simplified geometrical model of the measured degrees of freedom, constitute the experimental description of the system.

There is a range of complex and relevant problems in engineering that make use of modal data to solve a specific inverse problem, such as: numerical model updating and tuning, detection and identification of damage, identification of mechanical properties, force identification, and sensor optimization problems. The great advantage of using modal data is due to the amount and quality of global information intrinsic to the structure. This chapter is focused on discussing the use of modal data in order to solve a general inverse problem as well as discussing the peculiarities of selected applications.

## 3 The Frequency Response Function

Most of the parameter identification techniques and algorithms used in Modal Testing to obtain the modal parameters are based on a previous identification of at least one (usually several) FRFs of the system under investigation. Therefore, some basic definitions and equations involved in the modal identification methodology based on FRFs are briefly reviewed.

### 3.1 Frequency Response Function - SDOF systems

Some mechanical and structural systems can be idealized as Single Degree of Freedom (SDOF) systems. The theory for an SDOF system forms the basis for the analysis of a system with more than one degree of freedom (DOF). It also provides physical insight into the vibration of a structural system. We will use the SDOF system shown in Figure 2 that has a mass, a spring and a damper with either viscous or structural (hysteretic) damping.

For a harmonic force $f(t) = F(\omega)e^{j\omega t}$, the response of the system is another harmonic function $x(t) = X(\omega)e^{j\omega t}$ where $X(\omega)$ is a complex amplitude. Substituting into the equations of motion for viscous damping models, we can derive the ratio of the displacement response and the force input as:

$$H(\omega) = \frac{X(\omega)}{F(\omega)} = \frac{1}{k - \omega^2 m + j\omega c} \tag{2}$$

Briefly, the frequency response function is a transfer function that describes dynamic structural physical behavior (displacement, velocity or acceleration) as a ratio, in the frequency domain, to the input force. FRF depends on the intrinsic structural characteristics

operational environment. The procedure is usually called *Operational Modal Analysis*(Avitabile [2017], Covioli [2021]).

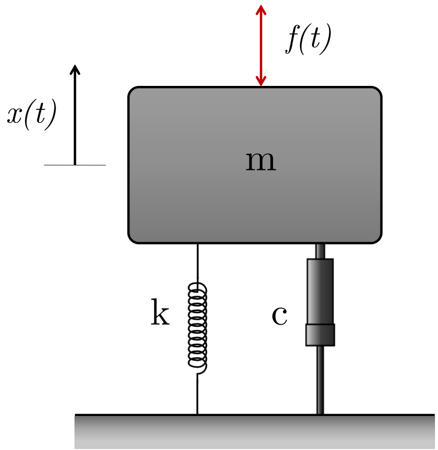



**Figure 2: Single degree of freedom (SDOF) system**

of mass, stiffness, damping and also the boundary condition (supports and bonds). Figure 3 shows the general idea of the FRF.

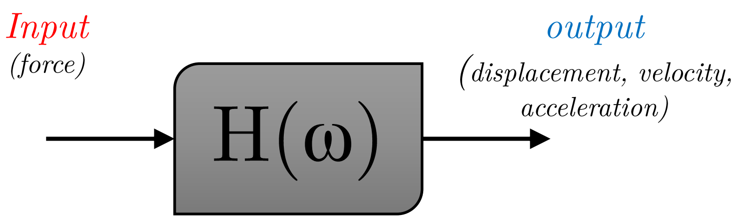



**Figure 3: Frequency response function**

The ratio written in therms of the dynamic displacement shown in Equation 2, often denoted as $\alpha(\omega)$, is defined as the *compliance* FRF of the system. Although defined as the ratio of the force and response, the FRF is independent of them. When damping is zero, the complex FRF function is relegated to a real function. The FRF is the main function on which modal analysis will depend. Although in theory the FRF is dictated only by the system, in reality the accuracy of measured FRF data is critical to the success of modal analysis.

The system FRF can also be represented in terms of the velocity or the acceleration of the mass. By replacing the displacement response $X(\omega)$ with velocity $\dot{X}(\omega)$ and acceleration $\ddot{X}(\omega)$, types of FRFs can be defined, usually called, respectivelly, as *mobility* ($Y(\omega)$) and *accelerance* $A(\omega)$) functions. The three functions may be written as

$$\alpha(\omega) = \frac{X(\omega)}{F(\omega)} = \frac{1}{k - \omega^2 m + j\omega c}, \tag{3}$$

$$Y(\omega) = \frac{\dot{X}(\omega)}{F(\omega)} = \frac{j\omega}{k - \omega^2 m + j\omega c}, \tag{4}$$

and

$$A(\omega) = \frac{\ddot{X}(\omega)}{F(\omega)} = \frac{-\omega^2}{k - \omega^2 m + j\omega c}. \tag{5}$$

It is clear that the three types of FRFs, $\alpha(\omega)$, $Y(\omega)$, and $A(\omega)$, are easily interchangeable. All three are complex functions of frequency. Their amplitudes follow:

$$|A(\omega)| = \omega|Y(\omega)| = \omega^2|\alpha(\omega)| \tag{6}$$

The phase difference among them remains constant at any frequency:

$$\theta_{A(\omega)} = \theta_{Y(\omega)} + \frac{\pi}{2} = \theta_{\alpha(\omega)} + \pi \tag{7}$$

The reciprocals of the three FRFs of an SDOF system also bear useful physical significance and are sometimes used in modal analysis. They are respectively known as Dynamic Stiffness (Eq. 8), Mechanical Impedance (Eq. 9) and Dynamic Mass (Eq. 10).

$$\frac{1}{\alpha(\omega)} = \frac{force}{displacement} \tag{8}$$

$$\frac{1}{Y(\omega)} = \frac{force}{velocity} \tag{9}$$

$$\frac{1}{A(\omega)} = \frac{force}{acceleration} \tag{10}$$

There are benefits to displaying the FRF in these different formats where some physical conclusion can be made, depending on the specific application. All of these responses are mathematically related (as seen in Equations 3 to 5) so by calculating one FRF, any of the other representations can be determined. In this sense, Figure 4 illustrate the different types of direct and inverse input/output FRF.

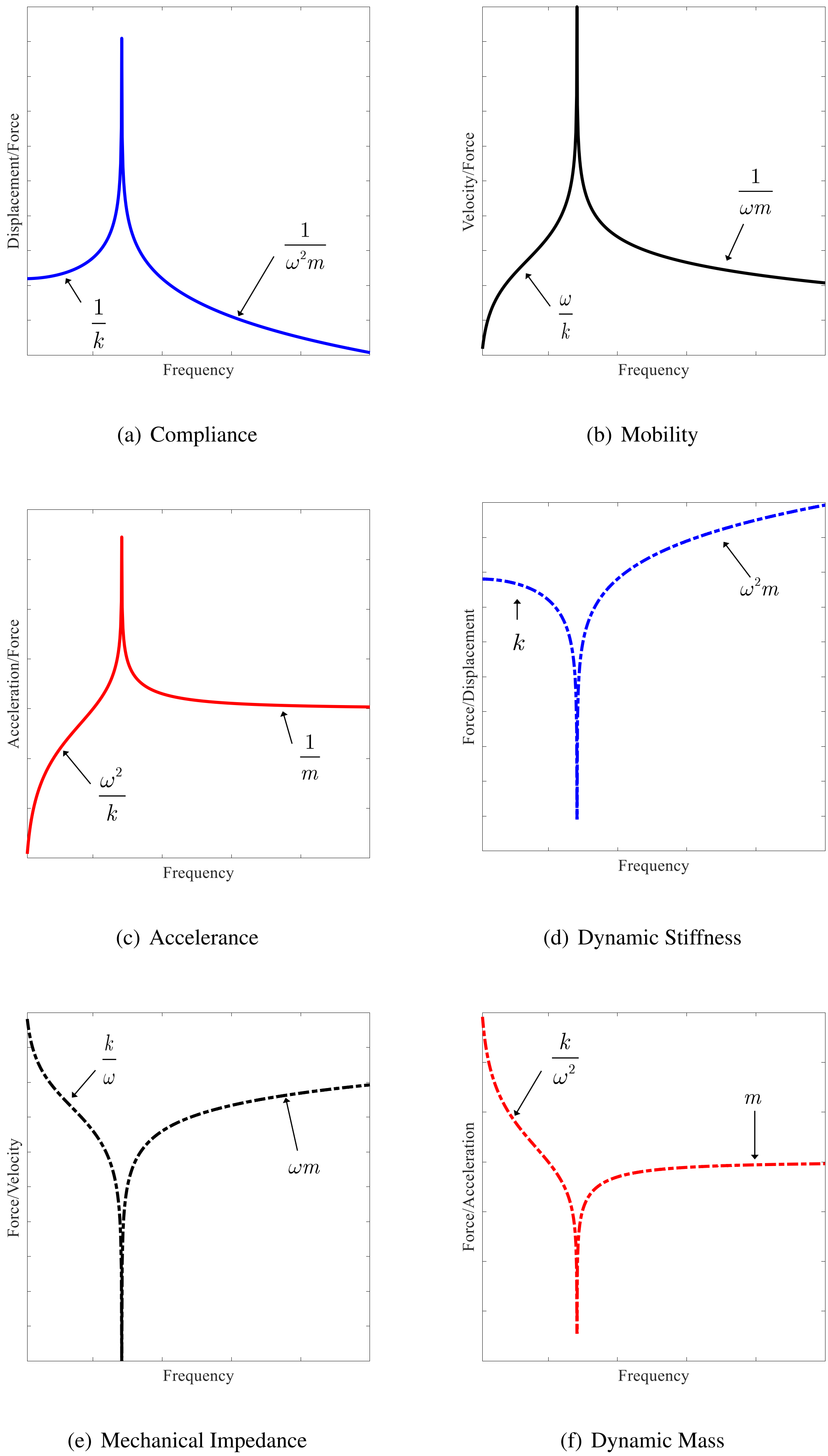
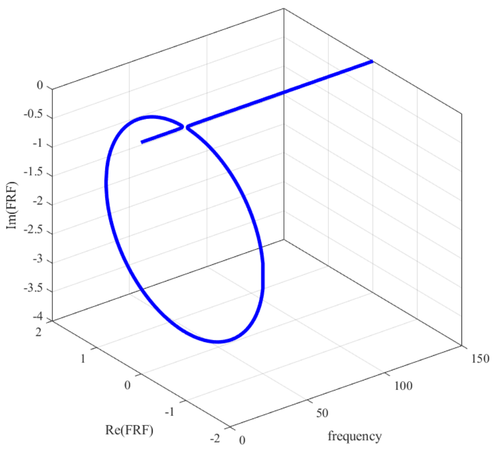
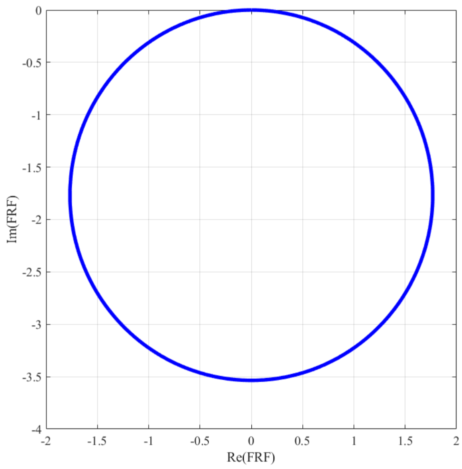



**Figure 4: The direct and inverse frequency response functions.**

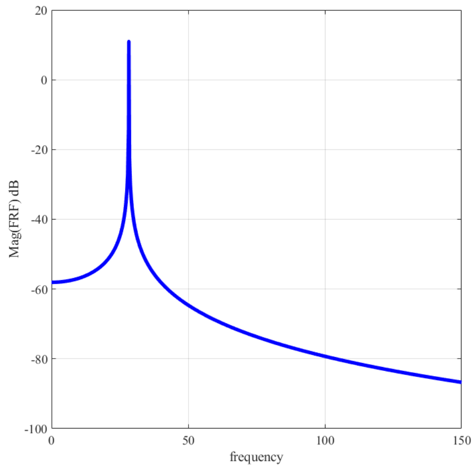
Graphical display of an FRF plays a vital role in modal analysis. A different graphical display highlights different information an FRF carries. Since experimental modal analysis often relies on curve fitting of FRF data, sound understanding of FRFs in graphical forms is imperative. In the following, we will show that even for the FRF of an SDoF system which seems to be analytically simple and trivial, much insight about the function can be gained by studying it in various forms of graphical display. We will use first the receptance FRF to begin our exploration of the graphical display. Since the receptance FRF is a complex function of frequency, it is impossible to fully display it using merely one two-dimensional plot. A three-dimensional plot of a receptance FRF of a typical SDoF system is shown in Figure 5.




(a) A three-dimensional plot of an FRF




(b) Nyquist plot of an FRF




(c) Magnitude of an FRF

**Figure 5: An overview of FRF for SDOF system.**

### 3.2 Frequency Response Function - MDOF systems

Next, a multiple degree of freedom system (MDOF) is considered. Similarly to the SDOF case, the equations of motion for a MDOF system can be derived assuming the following simplifications (Avitabile [2017]):

- the mass and or inertia properties of the system are modeled as lumped quantities
- the spring forces (or moments) are proportional to displacement (angular displacement).
- the damping forces (or moments) are proportional to linear velocity (or angular velocity).

Additionally, the derivation is restricted to the case where the physical properties are time invariant. Under these assumptions, the equations of motion for a $N$ degree of freedom linear, time-invariant system can be written in matrix form as

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = \{F(t)\}, \tag{11}$$

where $[M]$ is the mass matrix $(N \times N)$, $[C]$ is the damping matrix $(N \times N)$, $[K]$ is the stiffness matrix $(N \times N)$, $F(t)$ is the forcing vector $(N \times 1)$ and $x(t)$ is the vector $(Nx1)$ of displacements. These equations constitute the "spatial model", in the form of system of ODEs, which are usually coupled in the physical coordinates $x(t)$.

It is important to notice that, in structural dynamic problems, matrices $[M]$ and $[K]$ are quite often obtained from a finite element model. Matrix $[C]$ is usually introduced as a representation of localized damping devices in mechanical systems, as, for example, vehicle suspension systems, physical joints in multibody systems and other very specific mechanical problems. However, the damping matrix $[C]$ is seldom available for general, distributed parameter structural problems. The damping characteristics of such systems are introduced in the models from previous heuristic knowledge or, more frequently, from vibration experiments such as modal testing.

The Frequency Response Functions associated to Equation 11 may be also written in matrix form as $[H(\omega)]$, an $(N \times N)$ complex symmetryc matrix given by:

$$[H(\omega)] = \left(-\omega^2[M] + j\omega[C] + [K]\right)^{-1}, \tag{12}$$

where each individual term in matrix $[H(\omega)]$, denoted by $H_{rs}(\omega)$, represents the FRF related to the response at coordinate $r$ due to an excitation acting on coordinate $s$ of the structural system.

The representation of the system FRFs in Equation 12, despite its simplicity, may be computationally prohibitive for real-life, large structural models, which are easily obtained nowadays with the available FE modeling tools. A much more efficient approach is to introduce a change of coordinates intended to decouple the equations of motion - the *modal analysis* approach, presented in the following paragraphs (the time-dependency of the variables may be occasionally omitted for clarity).

The starting point is the equation for the free vibration of the undamped MDOF system, expressed by

$$[M]\{\ddot{x}\} + [K]\{x\} = \{0\}. \tag{13}$$

Assume that the response $x(t)$ is harmonic and has the general form

$$\{x\,(t)\} = \{X\}\,e^{j\omega t}, \tag{14}$$

where $\{X\}$ is an (Nx1) vector of time-independent complex response amplitudes. Substituting (14) into (13), we arrive at the algebraic eigenvalue problem

$$\left([K] - \omega^2\,[M]\right)\{X\} = 0. \tag{15}$$

The eigensolution of Eq.(15) yields N pairs of associated eigenvalues and eigenvectors, $\omega_r^2$ and $\{X_r\}$ , which are the natural frequencies (squared) and mode shapes of the system. The mathematical process of the eigensolution can be performed by means of direct and indirect techniques. For smaller matrices, the direct techniques decompose the set of equations to get all of the eigenvalues and eigenvectors. However, for complex structures, when the matrices get larger, as is the case for the large finite models, then indirect, iterative numerical techniques are used.

Due to the symmetric nature of the system matrices, the eigenvalues and eigenvectors are real quantities. Each eigenvector $\{X_r\}$ is a vector with $N$ real quantities that are only defined in relative terms, i.e, the direction of the vector is well defined, but not its magnitude. It is useful to represent the eigenvectors by their mass-normalized version $\{\phi_r : //www.overleaf.com/project/629239a6de6fa1d13e3e226bht\}$, imposing the normalization (Eq. 16).

$$\{\phi_r\}^T\,[M]\,\{\phi_r\} = 1 \tag{16}$$

The orthogonality properties of the eigenquantities with respect to the system matrices may now be written as shown in Eq. 17.

$$[\Phi]^T\,[M]\,[\Phi] = [I] \tag{17}$$

and

$$[\Phi]^T\,[K]\,[\Phi] = \lceil\omega_r^2\rfloor, \tag{18}$$

where $[\Phi]$ is the *modal matrix*, in which each column is a mass-normalized mode shape, and $\lceil\omega_r^2\rfloor$ is a diagonal matrix of the squared natural frequencies.

The orthogonality properties are useful to uncouple the system equations. Defining a coordinate transformation

$$\{x\,(t)\} = [\Phi]\,\{q\,(t)\}\,, \tag{19}$$

and substituting into Eq. (13) leads to

$$\{\ddot{q}(t)\} + \lceil\omega_r^2\rfloor\,\{q(t)\} = \{0\}\,. \tag{20}$$

Hence, the coordinate transformation leads to a set of uncoupled equations, corresponding to N independent undamped SDOF systems that can be solved separately.

In general, the damping matrix is not diagonalized by the coordinate transformation and the equations of motion for the damped system will remain coupled. However, under the useful assumption that the damping is *proportional*, i.e., the damping matrix $[C]$ can be written as a linear combination of $[M]$ and $[K]$, the damping matrix can also be diagonalized as

$$[\Phi]^T [C] [\Phi] = \lceil 2\zeta_r \omega_r \rfloor, \tag{21}$$

where $\zeta_r$ is the modal damping ratio for mode $r$. Next, take Eq. (19) into Eq. (11) and pre-multiply by $[\Phi]^T$, resulting in

$$\{\ddot{q}(t)\} + \lceil 2\zeta_r \omega_r \rfloor \{\dot{q}(t)\} + \lceil \omega_r^2 \rfloor \{q(t)\} = [\Phi]^T \{F(t)\}. \tag{22}$$

Equation (22) is the basis for a much more efficient forced response analysis of the original coupled damped MDOF system, now represented by N independent damped SDOF equations in the so-called modal coordinates $\{q(t)\}$. Also, it will lead to a more useful formulation of the system FRFs, more suitable for the development of parameter identification algorithms. After the necessary mathematical manipulation steps, the MDOF system FRFs can now be written as

$$H_{rs}(\omega) = \sum_{k=1}^{N} \frac{A_{rs(k)}}{\omega_k^2 - \ \omega^2 + 2j\zeta_k \omega \omega_k}, \tag{23}$$

with

$$A_{rs(k)} = \{\phi_k\}_r \{\phi_k\}_s, \tag{24}$$

where $\{\phi_k\}_r$ is the real entry at coordinate $r$ of mode shape $k$.

Figure 6 depicts a typical FRF of a MDOF system, illustrating the contributions of each single mode, combining the terms in the summation shown in Equation (23).

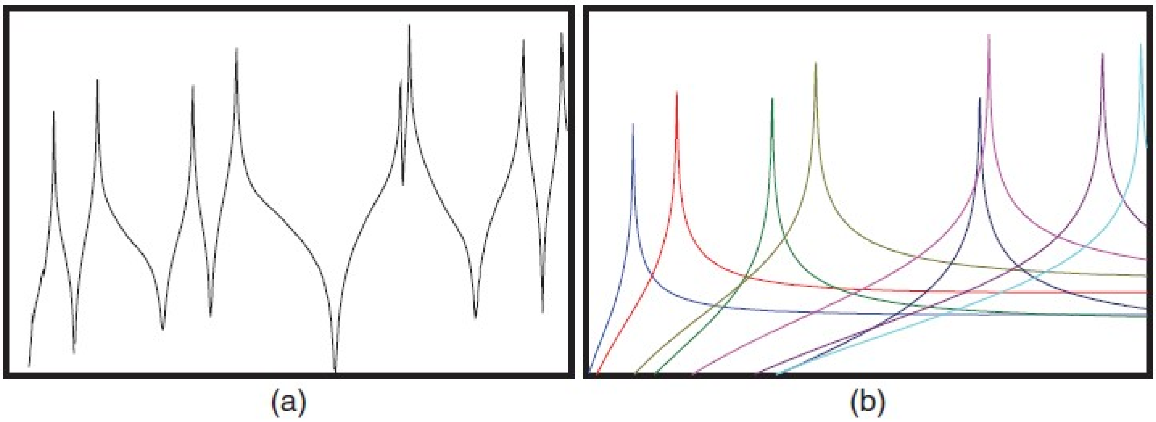
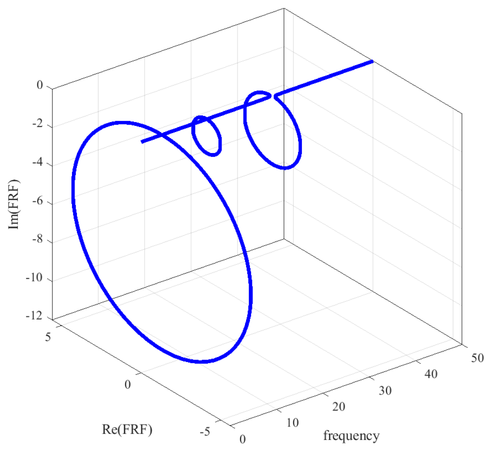
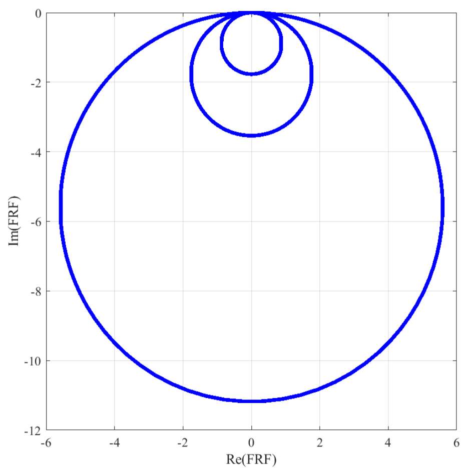



**Figure 6: Typical FRF for a MDOF system : (a) "summed" FRF ; (b) individual mode contributions (Avitabile [2017]).**

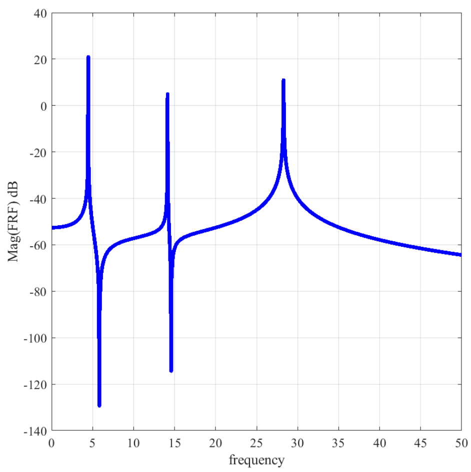
Complementary, Figure 7 shows the FRF plots for a multiple degree of freedom (MDOF) system. In this specific case, the numbers of modes in the shown frequency range are $N$ = 3.




(a) A three-dimensional plot of an FRF




(b) Nyquist plot of an FRF




(c) Magnitude of an FRF

**Figure 7: An overview of FRF for MDOF system.**

Such a three-dimensional plot is complete because it shows the true face of an FRF. It is, however, difficult to be used especially for modal analysis where characteristics such as resonance need to be visually available. From Figure 5, we can see that the 3-D plot, when projected to the frequency-real plane, becomes the real part of the FRF. Likewise, its projection to the frequency-imaginary plane gives the imaginary part of the FRF and that to the real-imaginary plane is the Nyquist plot. These plots (and their variations) highlight different aspects of the FRF. The need to investigate an FRF from a 2-D graphical manifestation gives rise to a number of different graphical presentations of it.

The wide range of modal parameter estimation strategies and algorithms developed over the years rely on some mathematical representation of the system FRFs similar to the one presented in Eq. (23). Slightly different derivations are needed, for instance, when considering more general damping models, which may result in complex mode shapes. Depending on the complexity of the modal test performed, the amount and quality of the acquired data, and the level of details of the desired modal model, one can choose identification procedures ranging from simple curve fitting for individual peaks in Fig. (6), circle fitting of Nyquist plots, line fitting of a broader frequency band including several modes or, for more complex demands, detailed algorithms that might include several FRFs for different excitation points and different measurement locations. The next section discusses some of the issues involved in the experimental identification of FRFs for modal parameter extraction.

### 3.3 Comments on the Experimental Identification of FRFs

To obtain experimental FRFs with good quality for a successful modal parameter estimation, several aspects of the modal testing procedure have to be carefully taken into account (Akers et al. [2020]). First, a thorough test planning must be conducted, including topics such as test objectives, selection of frequency range of interest, selection of measurement and excitation coordinates, selection of excitation devices and methods (impact or shaker, for example), selection of measurement transducers, definition of support points and apparatus, among others. It is very useful, whenever possible, to create a simplified theoretical/numerical model for preliminary simulations that can greatly facilitate the test planning process.

During the execution of the modal testing tasks, data acquisition and signal processing issues are of paramount importance. The setup of all electronic devices and connections that constitute the excitation and response measurement chains must be carefully and regularly inspected, to ensure that no extraneous influences are contaminating the acquired data. A good understanding of the basics of signal processing theory is also necessary. Issues such as sampling rates, windowing, filtering, averaging, coherence, etc. must be thoroughly considered. This knowledge is essential for the acquisition of data that is potentially free of the effects of spurious noise and data acquisition undesired phenomena, such as leakage and aliasing. For more detailed information on modal test planning and excecution, see Ewins [2000], Fu and He [2001], Peeters et al. [2004] and Avitabile [2017].

Once a set of good quality FRFs is available, it is time for the parameter estimation step. In theory, the FRFs of a MDOF system constitute a square (NxN) matrix of functions. In practice, however, only a limited number of functions is available. Therefore, it is important to make sure that the correct estimation technique is chosen for the case at hand.

Figure (8) illustrates an schematic classification of parameter estimation methods based on the fitting of the available FRFs. A local approach may be sufficient for smaller problems, using one FRF at a time to estimate natural frequencies, damping ratios and the amplitude of the mode shape for the corresponding coordinate. It is important to notice that the local method does not rely on a single FRF, since it would be impossible to estimate mode shapes. The local stands for analyzing one FRF at a time and combining the results in the end of the process. The global approach includes a set of FRFs obtained at different locations from a single excitation point. In this case, a "column" of the FRF matrix is fed to the estimation algorithm, resulting in the simultaneous estimation of the natural frequencies, damping factors and mode shapes. The polyreference approach is and extension of the global approach. Now, the algorithm can deal with data originated from two or more excitation points. Complex and sophisticated tests may be conducted with simultaneous multi excitation. It is more common, however, to obtain different columns of the FRF matrix in distinct test runs. Care must be taken to make sure that the same test conditions are reproduced for all the data acquisition runs.

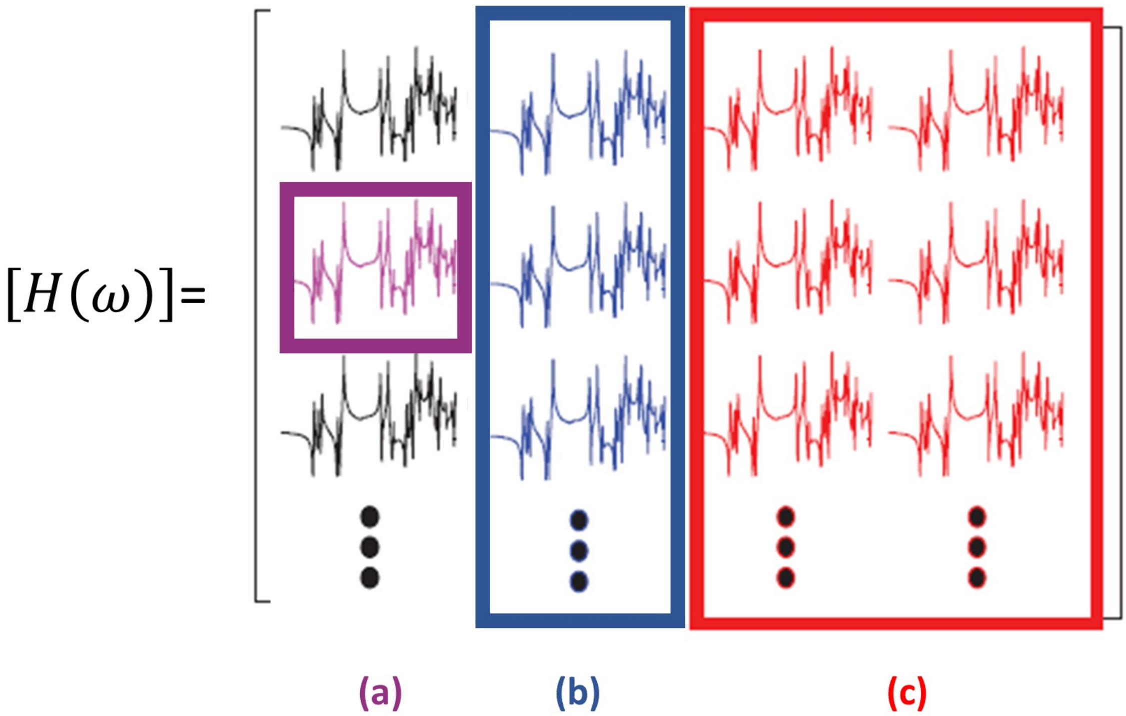
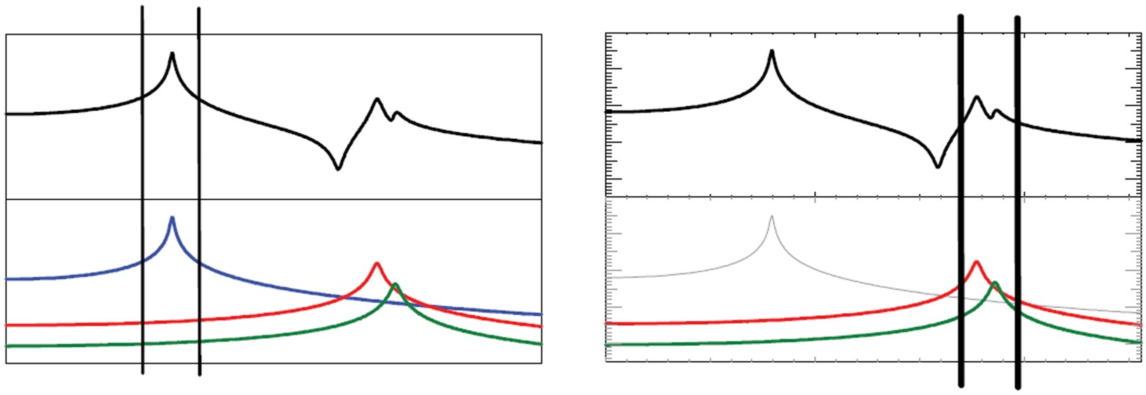



**Figure 8: Classification of modal parameter extraction aproaches: (a) Local curvefitting ; (b) Global curvefitting; (c) Polyreference curvefitting (adapted from Avitabile [2017]).**

Another important aspect to be considered during the parameter estimation step is related to modal density - the number of peaks in a given frequency band - and damping levels. Modes that are too close to each other may pose difficulties to less sophisticated estimation methods. When damping levels are high, it is sometimes impossible to distinguish two modes of close natural frequencies. Figure (9) schematically illustrates this very relevant point. The lower frequency mode in (a) is clearly isolated and a SDOF based method may be successful in estimating the modal parameters with good quality. However, the two peaks marked in (b) will certainly demand a more detailed, MDOF estimation algorithm. Two of the most popular methods found in the literature and implemented in commercial Modal Analysis software are the Least Square Complex Exponential method and the PolyMAX method (Avitabile [2017]).



(a) (b)

**Figure 9: Effect of mode spacing in estimation strategy : (a) SDOF curvefitting ; (b) MDOF curvefitting; (adapted from Avitabile [2017]).**

The topics discussed in the present section are obviously only a glimpse on the subject, intended to motivate the reader to a more detailed look at the comprehensive literature referenced at the end of the chapter. As a final comment about practical considerations of a modal testing, it is important to register that the execution of an experimental modal analysis campaign appears to be a challenge only suited for seasoned experimentalist with hundreds of lab hours in his/her résumé. This is certainly true for large complex structures, usually demanding a multidisciplinary team of professionals to get the job done within the required time and quality constraints (Carneiro et al. [1992]. However, a dedicated novice researcher can obtain very rewarding results with smaller problems, more typical in academic research.

## 4 Inverse Modal Problems

Nowadays, many researches have been focused on the development of several metrics based on the characteristics of static and dynamic (mainly) responses of mechanical systems. An inverse problem is a general strategy or methodology that is used to convert observed measurements into information about a physical object or system in which we are interested (Jorge et al. [2022]). There are several advantages to using modal data in inverse problems, since modal data usually reflect both global and local structural conditions with relevance (Gomes et al. [2018a], Magacho et al. [2021]).

An inverse problem based on modal parameters can be formulated basically using i) natural frequency, ii) damping factor or loss factor and iii) mode shapes. Additional formulations based on FRF and derived from direct responses such as modal curvature, energy and others are also widely used today. Modal parameters such as natural frequency are values that have a significant amount and quality of global structural information whereas mode shapes are more sensitive to local disturbances. The correct combination of these responses, depending on the formulation and application in question, can lead to reliable and accurate results in the identification of parameters.

Figure 10 briefly shows the general process of an inverse problem. Usually the experimental tests are carried out in order to extract the modal properties (natural frequencies, damping factors and mode shapes). These properties will feed the formulation of an inverse problem that is usually accomplished by minimizing an objective function. This objective function can be the norm (or error) between results measured and calculated by the optimizer. The importance of using optimization techniques, which play a fundamental role in the problem of parameter identification, is also noteworthy. Table 1 shows a list of the main types of objective functions used in most of the research work found in the literature.

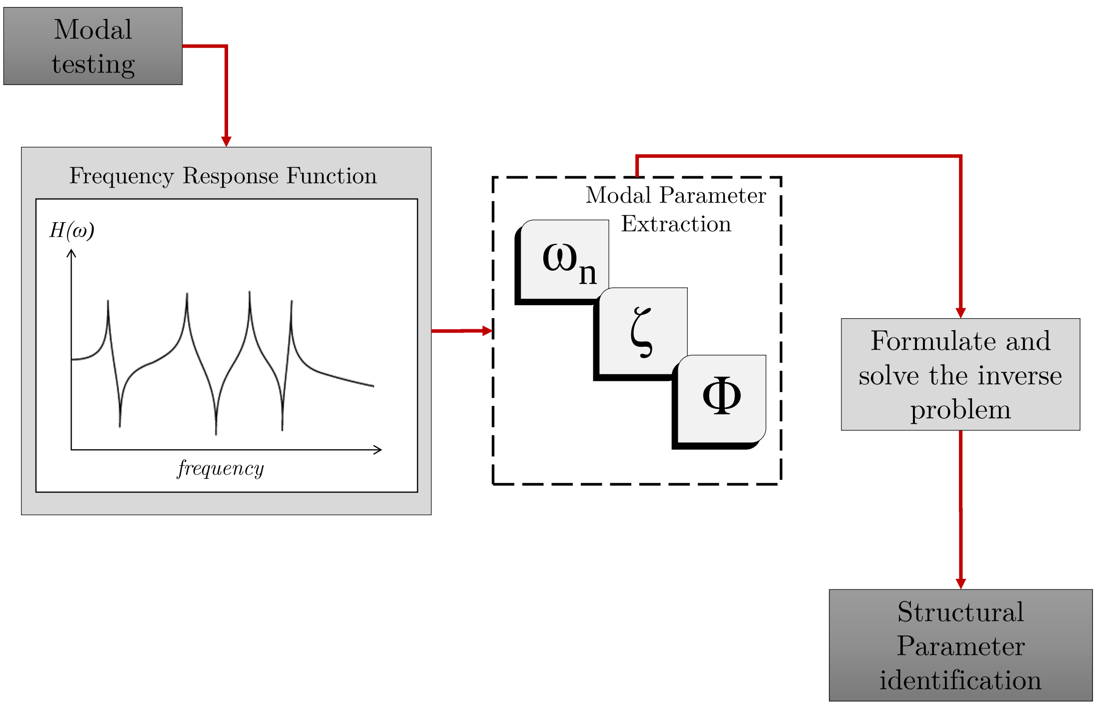



**Figure 10: A general flowchart of modal based inverse problems.**

In general, the most relevant, discussed applications and source of attention for several engineers and researchers are: damage detection and identification, sensor placement optimization (Gomes et al. [2019a], Gomes et al. [2020]), property identification, structural optimization (by maximizing the fundamental natural frequency) and identification of vibration forces. In this section, a brief discussion of the main contributions to each theme will be made, as well as the advantages and peculiarities of each formulation.

**Table 1: Systematic review of the parameters used in optimization in damage detection.**

| Author | Algorithm | Objective Function |
|---|---|---|
| Friswell et al. [1998] | GA | $J=\sum_{j=1}^{r}\left(\frac{\delta\omega_{mj}-\delta\omega_{aj}}{\delta\omega_{mj}}\right)^2+\sum_{j=1}^{r}(\phi_{mj}-\phi_{aj})^T(\phi_{mj}-\phi_{aj})$ |
| Yong and Hong [2001] | GA | $J=\sum_{i=1}^{nm}\left\{\left[\left(\frac{\lambda_i(\alpha)-\lambda_i^0}{\lambda_i^0}\right)^A-\left(\frac{\lambda_i^D-\lambda_i^U}{\lambda_i^U}\right)^E\right]^2+\left[(\phi_{ij}(\alpha)-\phi_{ij}^0)-(\phi_{ij}^D-\phi_{ij}^U)\right]^2\right\}$ |
| Jafarkhani and Masri [2011] | CMA | $J(\alpha)=\sum_{i=1}^{n}\left[\left(\frac{f_i^e-f_i^a}{f_i^a}\right)^2+\left(1-\frac{\lvert\phi_i^{eT}\phi_i^a\rvert^2}{(\phi_i^{eT}\phi_i^e)(\phi_i^{aT}\phi_i^a)}\right)\right]$ |
| Nanda et al. [2012] | PSO | $J=\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\left(\frac{f_i^m}{f_i^c}\right)-1\right)^2}$ |
| Mohan et al. [2013] | PSO | $J=\frac{1}{N}\sum_{q=1}^{N}\left(\sum_{a=1}^{R}\sum_{p=1}^{M}\frac{\lvert H_{ak}(\omega_p,\alpha)-H_{ak}^m(\omega_p)\rvert}{max(H_{ak}^m(\omega_p))}\right)$ |
| Suveges [2016] | DE | $J=\sum_{i=1}^{n}\left[(\sigma_{ix}^{sim}-\sigma_{ix}^{cal})^2+(\sigma_{iy}^{sim}-\sigma_{iy}^{cal})^2\right]$ |
| Cha and Buyukozturk [2015] | GA | $J=\sum_{i=1}^{ms}\sum_{i=1}^{el}\lvert\Phi_i^{dT}K_j\Phi_i^d-\Phi_i^{sT}K_j\Phi_i^s\rvert$ |
| Braun et al. [2015] | ACO | $J=\sum_{j=1}^{N}\sum_{i=1}^{N_t}\left[x_j^{Mod}(K,t_i)-x_j^{Exp}(K,t_i)\right]^2$ |
| Gomes et al. [2016a] | GA | $J=\sqrt{\frac{1}{N}\sum_{i=1}^{n}\left(1-\frac{\omega_i^d}{\omega_i(\vec{X})^c}\right)^2}$ |
| Gomes et al. [2016b] | GA | $J=\sqrt{\frac{1}{N}\sum_{i=1}^{n}\left(1-\frac{\omega_i^{real}}{\omega_i(\vec{X})^{model}}\right)^2}+\sum_{i=1}^{n}\left(\ddot{x}^{real}-\ddot{x}^{model}\right)^2$ |
| Vo-Duy et al. [2016] | DE | $J=\sum_{i=1}^{nm}\frac{\lVert\Phi_i^d-\Phi_i^d(x)\rVert}{\lVert\Phi_i^d\rVert}$ |

### 4.1 Structural Health Monitoring Problems

The damage can be seen as a change in the geometric or physical properties of a material. A structure, in the presence of a damage, will have a mechanical behavior (parameter) that will not resemble an intact structure. These parameters are directly affected by the variation in the physical (modal) properties of the structure (mass, rigidity and damping). When all is said in done, damage may cause local changes in the mass and/or stiffness of the structure and, as an outcome, alter its attributes.

The structural monitoring by itself does not find the solution to structural problems, however, according to Worden et al. [2009], if it is used in conjunction with the development of new computational technologies and mathematical tools, it opens the possibility of solving optimization problems associated with Structural Health Monitoring (SHM) and finding the best solution.

According to Sohn et al. [2003], many damage detection methods reviewed attempt to identify damage by solving an inverse problem, which often requires the construction of analytical models. This reliance on an earlier expository model, which is regularly unverifiable and not completely approved with test information, makes these methodologies less alluring for specific applications.

There are various damage indicators (Gomes et al. [2018b]). Several are constructed from vibration data, either in the time and/or frequency domain. These indicators have proved to be efficient, but there are still areas that need to be improved. Many indicators present sensitivity problems, need a reference state and do not present the probability of detecting false alarms, reducing their reliability.

According to Gopalakrishnan et al. [2011], the SHM methodology offers a complete platform to predict failures before they occur, for this, it is necessary that some properties of the structure under study are provided and the configuration and detection parameters are established, it should also be indicated if the monitoring will be done in part or continuous time. The SHM methodology, once proven reliable, contributes to the premature detection of damages that have led to the collapse of the structure and is an indicator for the scheduling of preventive actions. The field of application of this technique varies from civil structures of the aeronautics and space industry. Some of the main advantages of the SHM application are: i) to minimize the time of application of maintenance actions and stops due to serious failures, ii) the information collected provides valuable data that can be used to adjust the structure mode of operation or even to optimize the design in future improvements and, iii) allow a reconditioning of maintenance actions based on operating and performance conditions and not in periodic maintenance actions, that (probably) may be unnecessary.

Damage is the main cause of structural failure and occurs frequently in mechanical structures. In recent decades, special attention has been given to methods of detecting damage at an early stage to avoid sudden failure of structural components. More specifically, the monitoring of structural integrity based on the vibration of structures has been the focus of attention of many researchers in order to obtain efficient tools of great importance for the civil, aeronautical and mechanical engineering communities. In addition, many research fronts have been focused on the development of reliable damage indicators that allow, in addition to detecting damage, identifying it in terms of location.

One of the first published studies of relevance was developed by Adams et al. [1978]. The authors have developed a non-destructive method of assessing structural integrity based on vibration measurements. It was shown that the vibration measurements made in the structure (receptance function) can be used, together with a suitable theoretical model (reference), to indicate the location and magnitude of damage, in a one-dimensional model.

At the same time, Cawley and Adams [1979] realized that a certain state of damage can be generated by a reduction in stiffness or by an increase in structural damping. Changes in stiffness, whether local or distributed, lead to changes in the natural frequencies of the structure in question. Further, since the stress distribution across a vibrating structure is non-uniform and is different for each of the natural frequencies, any localized damage would affect each mode individually and differently, depending on the particular location of the damage. Thus, the measurement of modal data of a structure in two or more stages of its life offers the possibility of locating damages in the structure. If a set of dynamic responses is measured before the structure is in service, measurements of those responses can be used to determine if the structure still meets certain operational criteria.

Several researchers develop strategies based on inverse problems in order to identify damage to mechanical structures. The great advantage of using modal data is due to the ease of experimental tests in loco and the quality of the data. Natural frequency data is poorly sensitive to climate change and external factors. In addition, natural frequencies are able to extract the overall structural situation with quality, being a good metric for detecting (whether or not there is damage). However, damage identification strategies can be improved with the aid of vibration modes. Mode shapes are more sensitive to local structural variations. However, mode shapes are sensitive to external noise. A good damage identification strategy can make use of modal parameters combined with the aid of advanced signal processing and acquisition techniques.

In summary, the development of an efficient structural monitoring technologies aims to provide safety and cost savings. However, the number of practical applications of these technologies is still finite. This is mainly due to the complexity of possible damage scenarios and the high performance requirements of the identification methods employed. The study discussed in this section refers mainly to the relationship between these two aspects, in order to reach a specific level of maturity.

### 4.2 Sensor Placement Optimization

The basic problem of fault and damage detection is to deduce the existence of damage to a structure from measurements made on distributed sensors. It is known that the quality of these measurements, that is, the quality of the structural monitoring, is largely dependent on where the sensors are located in the structure. Cost and practicality issues prevent the instrumentation of all points of interest in the structure and lead to the selection of a smaller set of measurement sites (Barthorpe and Worden [2009], Gomes and Pereira [2020]). The objective of this study is to indicate the problem of sensor placement optimization (SPO) and to describe some methods that have been investigated for its solution besides proposing an alternative optimization of the optimal positioning of these sensors. The following discussion focuses on sensor optimization techniques based on the dynamic structure.

Traditionally, a successful sensor distribution has been heavily dependent on the knowledge and experience of those conducting experimental tests. Practical methods, for example, by choosing sites close to anti-knots of low-frequency vibration modes, are combined to create coherent sensor distributions (Barthorpe and Worden [2009]). However, for a lot of the times, a single mode of vibration does not have enough information on a damaged structural state, being necessary the use of a set of modes. Therefore, a distribution in the anti-nodes would not be feasible for a predefined number of sensors.

According to the theory discussed in (Barthorpe and Worden [2009]), the objective of sensor positioning can be stated as the need to select a subset of measurement locations from a large finite set of locations, so as to represent the system with the highest possible accuracy using a limited number of degrees of freedom accessible. This can be seen as a three-step decision process:

- Number of sensors - How many sensors need to be placed in the structure to allow a satisfactory dynamic test?
- Sensor positioning optimization - Where should these sensors be located to obtain more accurate data?
- Evaluation - How can the performance of different sensor configurations be measured?

In general, on the first aspect, the minimum requirement for the system to be observable is that the number of sensors required can not be less than the number of modes to be identified, with an upper limit usually imposed by the cost or availability of the equipment. The second aspect is the area that has attracted the most interest. For the limited number of available sensors, the problem is the development of a suitable sensor positioning performance measurement to be optimized and the selection of an appropriate method. Some approaches require a single calculation to be performed, some are iterative, and many others take the form of an objective function to which an optimization technique must be applied. The third and last aspect includes several possibilities for evaluating the performance of chosen sensor sets. In this work, preferably the positioning item will be approached, where a pre-defined number of sensors will be fixed.

In addition, the sensor placement issue attracts a lot of attention from academia and industry, especially due to the growing number of large instrumented monitoring structures over the last decade. This is due in part to economic reasons, to the high cost of data acquisition systems (sensors and their supporting instruments), partly because of the limitations of structural accessibility (Rao et al. [2014]).

The set of degrees of freedom measured in most large structures, usually the shifts of the low frequency modes, provide enough information to describe the dynamic behavior of a structural system with sufficient accuracy to allow its structural state and/or modifications determined in an effective manner. Thus, the fundamental problem is how many and which degrees of freedom must be considered in the process of structural identification. To solve this problem, economic factors that require a limited number of sensors to be placed in accessible locations in the actual structure (Rao and Anandakumar [2007]) should be duly taken into account. Still, according to Rao and Anandakumar [2007], it is crucial that the sensors are located in the most advantageous locations. Otherwise incomplete modal properties will be measured and an accurate assessment of structural health monitoring will be impossible.

Figure 11 shows a general flowchart of a SPO process using an inverse problem. The flowchart starts with numerical structural modeling (discretizing the largest amount of

DOF possible). After obtaining the model, a given objective function $J$ is chosen in order to be minimized/maximized. The optimization of this function can be performed using different algorithms (gradient-based or gradient-free) depending on the complexity of the SPO problem. If a given stop criterion is satisfied, the optimal layout of the $N$ predefined sensors is obtained.

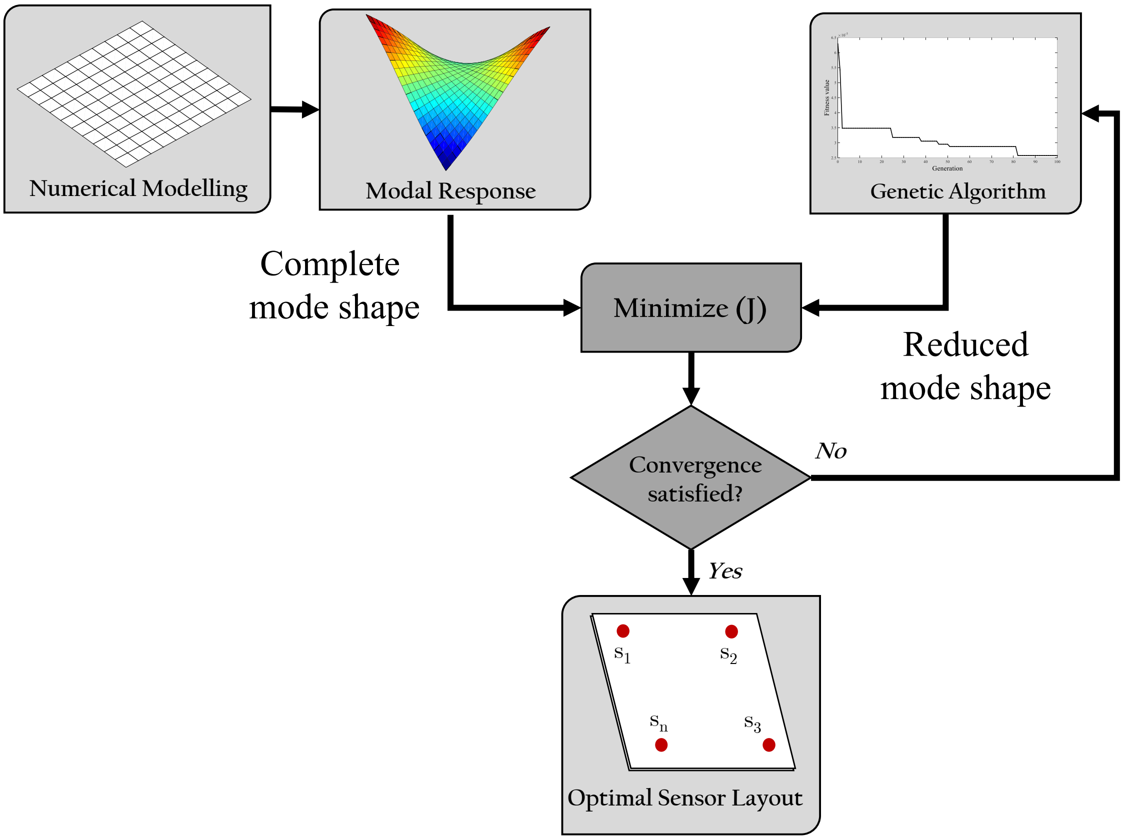



**Figure 11: General methodology for SPO based mode shape.**

### 4.3 Identification of Material Properties

Modal responses can also be part of formulating inverse problems in terms of identifying properties. This theme has great relevance because it allows non-destructive (vibration) tests to be conducted in place of traditional destructive tests (such as tensile, compression, bending tests, etc.). In addition to the identification of unknown properties, this formulation strategy can also be used to update and adjust experimental-numerical models.

Modal testing has the potential to provide the basis for rapid, inexpensive characterization of both elastic and viscoelastic properties of composites for design and manufacturing. Knowledge of elastic properties is, of course, required in design, but measurement of elastic properties during manufacturing offers the potential for improvements in quality control as well (Gibson [2000]).

The strategy of identifying mechanical properties by means of an inverse modal problem has its importance in isotropic materials. However, its great justification occurs in composite materials. The behavior of composite structural systems in which parameters are variable or not perfectly known is still a challenging problem. The most available models to investigate the behavior of composite structures assume an effective homogenized set of material properties. These models fail to capture the true behavior of the wide variety of such structures exhibit significant inherent uncertainty in material parameters (Sepahvand and Marburg [2014]).

In addition, according to Gibson [2000], not only does the material property characterization add to the cost of manufacturing, but the complexity of the test procedures and the test equipment is such that small and medium-sized composite fabrication shops generally have to rely on outside testing laboratories to do the characterization. Such a separation of the manufacturing and testing functions precludes an on-line evaluation of the properties, which is needed for optimization and control of the manufacturing process.Ideally, the composite characterization system should be fast, inexpensive and capable of providing feedback.

## 5 The Use of Modal Data in Inverse Problems

The central thought for vibration-based damage detection is that damage causes a modification in the mass, stiffness and damping structural matrix. These modifications will also cause recognizable changes in modal properties (natural frequencies, modal damping, and mode shapes). In this section, some classical and modern methodologies based on vibration signals will be described and discussed. Modal experimental analysis is an efficient tool to detect and identify damages, especially delamination in composite materials.

### 5.1 Natural Frequency-Based Methods

The presence of damage or structural deterioration causes changes in the natural frequencies in the structure. The most useful damage-finding methods (based on dynamic tests) are probably the ones that use changes in resonance frequencies (natural frequencies), because they are easy to obtain and are reliable metrics. Lifshitz and Rotem [1969] present what may have been the first article to propose damage detection through vibration measurements. They analyze the change in the dynamic modules, which may be related to the frequency change, indicating structural damage.

The effect of delamination on the natural frequencies of laminated composite beams has been investigated by Valdes and Soutis [1999]. The use of vibration at higher frequencies allows the identification of delamination occurrence. Additional setups can be also used in addition with modal frequency data. Ling et al. [2004] developed fiber-optic sensors to measure natural frequency in composite structures. The advantage of using fiber-optics is that they can be incorporated in laminated composites structures allowing real-time damage detection. LeBlanc et al. [1992] described how a network of embedded optical fibers could be used to detect and monitor damage (in particular, impact damage) within composites.

Also, as stated by Negru et al. [2015], transversal cracks in composite structures affect their stiffness as well as the natural frequency values. For a given crack, irrespective of its depth, the frequency drop ratio of any two transverse modes is similar (de Assis and Gomes [2021]). This permitted separating the effect of damage location from that of its severity and to define a Damage Location Indicator as a function of the square of the normalized mode shape curvatures, according to the authors.

Natural frequencies are good damage metrics, not only for delamination, but also for impact analysis. Velmurugan and Balaganesan [2011] studied the natural frequency and damping factors before and after impact as well the effect of damage on natural frequency and damping factors. It was shown that natural frequencies increase with the impact velocity.

### 5.2 Damping-Based Methods

According to Cao et al. [2017], typical dynamic properties refer to modal parameters: natural frequencies, mode shapes, and damping (or loss factor). Among these parameters, natural frequencies and mode shapes have been investigated extensively for their use in inverse problems such as damage characterization. Equally important, the use of damping as a dynamic property to represent structural damage has not been comprehensively elucidated, primarily due to the complexities of damping measurement and analysis. Furthermore, with advances in instrumentation and analysis tools, the use of damping as index (or objective function) becoming a focus of increasing attention in the modal-based inverse problems (especially for SHM community).

The main advantage of damping property is that damping has greater sensitivity for characterizing damage than natural frequencies and mode shapes in various applications, but damping-based damage identification is still a research direction “in progress” and is not yet well resolved (Cao et al. [2017]). According to Curadelli et al. [2008], damping is a promising damage indicator in structural health monitoring because it has more sensibility to damage than the natural frequency

Structural damping can be identified in both the time domain and frequency domain responses. The main methodologies to damping identification are: *i)* half-power (-3dB) method, *ii)* SDOF time response adjustment, *iii)* Circle fit in Nyquist plot, *iv)* logarithmic decrement method, *v)* Ibrahim time domain method, *vi)* wavelet transform, *vii)* Hilbert transform and many others.

From previous discussion, it can be noted that, whereas damping could be assumed negligible in free vibration analysis, it plays a crucial importance in forced response and must be quantified. In general, the overall damping of a system (SDOF or MDOF) is usually the most difficult parameter to obtain (Adams and Askenazi [1999]). Table 2 shows some selected representative damping ratios.

Among all damping models, viscous damping is most widely used because of its convenience in structural design. In fact, the viscous damping hypothesis has a serious drawback. The energy loss per cycle depends on the frequency; however, this result is inconsistent with many experimental results (Chopra, 2001). Another damping assumption, hysteretic damping, can more accurately describe the energy dissipation. Although it is difficult to translate the hysteretic damping mechanism into the time domain, it is promising to use the hysteretic damping assumption in the frequency domain (Pu et al. [2019]).

Apart from the shifts of the natural frequencies a change takes place also for the damping of the structure. Modal damping ratios would include the hysteretic damping resulting from non lnear phenomena and yielding of the structural members. Hence, damping ratios are very sensitive indices of the extent of inelastic deformation that takes place during

**Table 2: Damping constants values for different systems (Adams and Askenazi [1999]).**

| System | Damping Ratio ($\zeta$) |
|---|---|
| Metals (in elastic range) | $<$0.01 |
| Continuous metal structures | 0.02 to 0.04 |
| Metal Structure with joints | 0.03 to 0.07 |
| Aluminum/Steel transmissions lines | $\sim$0.0004 |
| Small diameter piping systems | 0.01 to 0.02 |
| Large diameter piping systems | 0.02 to 0.03 |
| Auto shock absorbers | $\sim$0.30 |
| Rubber | $\sim$0.05 |
| Large buildings during earthquakes | 0.01 to 0.05 |
| Prestressed concrete structures | 0.02 to 0.05 |
| Reinforced Concrete Structures | 0.04 to 0.07 |

physical structural disturbance (Kouris et al. [2017]).Equally important, according to Sehgal and Kumar [2021], in many fields, damping identification provides more sensitive approach to characterize structural damage compared to natural frequencies and modal assurance criterion (MAC) values, but damping-based identification of structural damage needs to be explored further.

In the SHM field, according to Souza et al. [2019], several damage identification approaches are based on computational models, and their diagnostics depend on the set of modelling hypotheses adopted when building the model itself. Among these hypotheses, the choice of appropriate damping models seems to be one of the key issues. Normally, damping models may not provide an increase of knowledge of some unknown parameters when damping rates are lower than 1%. Equally important, real modal properties or real FRF data without considering damping will cause failure to detect the real damages. It is well known that damping mechanism in a real structure is very complicated, and it remains the least-known aspect compared with stiffness and mass. However, the damping correction in a damage detection or FEM updating study will improve the physical parameter detection accuracy (Pu et al. [2019]).

### 5.3 Mode Shape and Mode Curvature-Based Methods

In addition to the eigenvalues, the eigenvectors (vibration modes) are fundamental and important data in the evaluation of damages in composite materials. In general, natural frequencies are excellent data to detect damage; that is, whether or not there is structural damage. The evaluation of the modes of vibration is more local; that is, these data allow the detection of the location of damage. Fu et al. [2016] presented a two-step approach based on modal strain energy (MSE) and response sensitivity analysis to identify local plate damage. The local damage was simulated by a reduction in modulus of elasticity. The important point is that a method to weaken the “neighborhood effect” has been proposed to reduce false alarms in the location of damages, as these are one of the major challenges facing the SHM community today. Numerical examples were then conducted to illustrate the efficiency of the proposed method, and thus, damages could be successfully identified even under the effect of measurement noise. There are some other new developments on the two-step approach or modal strain energy that can be seen in the studies of Hu et al. [2006], Yang et al. [2016] and Moradipour et al. [2015].

Under this approach, Hu and Wang [2009] and Hu and Wu [2009] showed the detection of surface cracks in a laminate using the modal strain energy method. First, the properties of the material were unknown and were obtained using an inverse method through finite element analysis and experimental modal analysis. Modal displacements were used to calculate the modal deformation energies and a damage index was defined by the authors employing the fractional modal deformation energy of the laminate before and after the damage. Consequently, the damage indices obtained from global and local measurements were able to locate the surface crack in the laminate. In this proposed method, only a few modes of vibration were required, and the authors concluded that the method has a relatively low cost and flexibility in measurement, allowing a non-destructive evaluation and feasibility of real-time detection in laminated structures.

It is also noted that Zhang et al. [2013] presented a new method of detecting damage to plates based on the curvature of the frequency shift surface (FSS). Unlike other vibration properties commonly used as vibration modes that may present low accuracy in practice, the proposed method was used as a way to overcome this problem. In addition, it has been found that local damage will only cause local change in FSS, which can be considered an abnormality, since the curvature of the FSS of an intact plaque is smooth, according to the assumption that intact plaque structures are often homogeneous. Compared with traditional methods, the method proposed by the authors has been shown to be more sensitive and accurate in identifying damage.

Techniques based on vibrations and modal data, although they have existed for some decades, are still widely used to the present day. Manoach et al. [2017] presented a time domain method based on Poincaré maps of the motion of a beam subjected to harmonic loading. The proposed damage index is based on the Poincaré maps of the forced response of the healthy and damaged structures. Numerical and experimental results confirmed that the Poincaré map-based method can be successfully used to detect and locate damage. Yang et al. [2017b] presented a Chebyshev pseudo spectral modal curvature formulation for damage detection in beam-like composite structures. The proposed method was developed to overcome the wrap-around problem of the Fourier spectral modal curvature and the authors stated that the damage detection performance is better than the Fourier. Similar results can also be seen in the research of Yang et al. [2017d], Yang et al. [2017c] and Yang et al. [2017a]. Recently Yang et al. [2018] dealt with the problem of vibration health monitoring based on Poincare map method, which has been numerically and experimentally verified as an effective tool in damage assessment. The authors concluded that the performance of the Poincare maps method depends on the selection of excitation, also stated by Yang et al. [2017e].

Mode shapes and their corresponding spatial derivatives can also be used in the formulation of inverse methods for damage assessment using local/global modal strain energy information. Domingues et al., Domingues [2019] investigated the applicability of this approach in detecting and localizing damage areas in aluminun-aluminum honeycomb sandwich panels used in the constructions of the first Brazilian Geostationary Satellite. The method combines a tuned FE model of the structural element, which provides the stiffnes matrix of the undamaged system, and the experimental natural frequencies and mode shapes of the potentialy damaged panel, obtained from a modal testing procedure. The method showed very promising results, successfully identifying stifness variations as small as 5% in regions of the order of 10 % the total area of the panel.

A nondestructive identification method for composite beams damage was explored by He et al. [2017]. The authors based their work on the curvature mode difference (CMD) as damage index. By combining experimental modal analysis with finite element simulation, the authors were capable of detecting and identifying damage sites. Similarly, Zhou et al. [2018] studied curvature mode shapes for damage identification in laminated composite plates. Numerical simulations demonstrate that the Continuous Wavelet Transform (CWT) is more sensitive to damage identification. Indeed, finite element simulation can be considered as very powerful tool in SHM systems. Some related works using FEA and SHM damage detection problems can be seen in Alaimo [2018], Chronopoulos [2018], Manoach et al. [2017] and Kefal et al. [2017]

### 5.4 No-Baseline Methods in Inverse Problems

Many methods used by the SHM community need to know the *a priori* condition or structural response of an intact or healthy structure and then, with continuous monitoring, be able to detect damage from changes in a known signal. This strategy works very well and has been used to this day. On the other hand, new methods that do not need a baseline (no-baseline methods) have been emerging.

Trendafilova et al. [2015] presented a Vibration-Based Structural Health Monitoring (VSHM) technique which is developed and applied for delamination assessment in composite laminate structures. The work suggests that the mutual information is a measure for nonlinear signal cross correlation. The mutual information between two signals measured on a vibrating structure is suggested as a damage metric and its application for the purposes of damage assessment is discussed and compared to the application of the traditional linear signal cross-correlation. The authors modeled the damage as a local stiffness reduction (delamination modeling) and stated that the developed damage metric is efficient for the purposes of delamination diagnosis in a composite laminate beam. The same results for laminated plates can be seen in the work of Garcia et al. [2015]. Going further, in the same context, Garcia and Trendafilova [2014] introduced a methodology for structural vibration analysis and vibration-based monitoring which utilizes a special type of Principal Components Analysis (PCA), known as Singular Spectrum Analysis (SSA). The method was applied in a composite laminate beam based on the decomposition of the frequency domain structural variation response using new variables, the Principal Components (PCs). Experimental results demonstrated that different damage scenarios can be clustered and clearly distinguishable.

Due to the above, other unconventional methods making use of structural dynamics are proposed by some researchers (Bovsunovsky [2018]; Park and Oh [2018]; Tributsch and Adam [2018]; Yin et al. [2017]; Abdeljaber et al. [2017]; Zhang et al. [2017]; Souza and Nóbrega [2017]; de Azevedo et al. [2017]). It is clear that there is a need in the development of effective structural monitoring techniques so that the safety and integrity of the composite structures can be improved. Qiao et al. [2007] evaluated dynamic-based damage detection techniques for laminated composite plates using intelligent piezoelectric materials and modern instrumentation such as the Scanning Laser Vibrometer (SLV). The study, aimed at the detection of delamination, made use of the measurements of the curvature of the modes of vibration, measured indirectly and directly. The implementation of the algorithm by the authors was successful in the detection of laminated plate delamination, demonstrating that the dynamic-based damage detection approach using curvature is a viable technique for the monitoring of composite structures.

Equally important, Viglietti et al. [2018] presented studies on free vibration analysis of some aircraft composite structures with a tapered shape which were analyzed using a 1-D Carrera Unified Formulation (CUF) model, considering different types of damage. The authors' results demonstrated that their approach provided an accurate solution for the free vibration analyses of complex structures and is able to predict the consequences of a global or local failure of a structural composites components.

## 5.5 Hybrid Methods

As discussed in the previous paragraphs, natural frequencies are excellent overall damage detection metrics while vibration modes are excellent local metrics for identification. Methods that implement both data can be considered more robust and in some cases, more efficient.

In the face of this reality, in recent years, several research fronts have been joining efforts to design better damage rates based on modes of vibration, since these are more effective in locating structural damage. Citing a similar case, Kim et al. [2003] pointed to a methodology to locate and non-destructively estimate the size of damage in structures, for which only some natural frequencies or some modes of vibration are available. First, the authors devised a method of natural frequency-based damage detection. An algorithm was then developed to locate damages by alterations in natural frequencies, being able to estimate crack size from frequency disturbances. Next, a method of damage detection based on vibration modes is described. Both methods are evaluated for several numerically simulated damage scenarios, for which two natural and mode shapes are generated from finite element models. The results of the analyses indicated that the two methods correctly located the damage, but the methodology based on the modes presented greater precision in the identification of cracks.

A hybrid technique proposed by Lopes et al. [2006] was able to identify delamination in composite plates. The damage was identified using a technique that the measured curvature differences before and after impact damage and also natural frequencies. Based on the hybrid technique presented, it was possible to identify internal damage on a laminated composite plate. Equally important, Gomes et al. [2016b] and Gomes [2016] applied an inverse optimization problem in order to detect and identify circular holes and delamination in CFRP plates. The cost function in the optimization procedure was a built in function of natural frequencies and mode shapes. The authors realized that when greater importance (weight) is given to the portion of the mode of vibration, better results are obtained. In fact, as discussed in this review, modes of vibration are better at identifying damages while natural frequencies are more robust and are excellent detection metrics (whether or not there is damage).

Eraky et al. [2015] proposed a procedure based on comparison of modal strain energy for different structural conditions from a damage index (DI). The DI was constructed based on modal data from natural frequencies and mode shapes, respectively. It was noticed that both the experimental and numerical results showed good agreement in identifying damages in flexural structural elements. In the same way, Owolabi et al. [2003]

measured changes in the first three natural frequencies and corresponding amplitudes of the measured acceleration frequency response function were used as a damage detection scheme in beams. Since the frequencies and amplitudes depend on the crack depth and location, these values can be uniquely determined by an inverse problem, thus identifying the damage location. Related research can be found in the literature about damage detection in composite materials using hybrid methods (Qiao et al. [2007]; Ručevskis et al. [2009]; Fox [1992]; Zhang et al. [2010]; Ashory et al. [2017]).

### 5.6 Frequency Response Function-Based Methods

In addition, safety and economic aspects are the main motivations for increasing research on the monitoring of structural integrity. Since the damage changes the dynamic characteristics of a structure, its modal properties (natural frequencies, damping and modes of vibration), several techniques based on experimental modal analysis have been developed in recent years. Not only are natural frequencies used as damage metrics, but also the use of frequency response functions (FRF) are widely explored by several researchers. As an example, the FRF curvature method was proposed by Sampaio et al. [1999], based only on measured data without the need for any modal identification. In the authors' work, the method was described theoretically and compared to two, more referenced methods in the literature. The results showed that the FRF curvature method obtained good results in the detection, localization and quantification of damages, although this last item still needs more attention. Its main advantage is its simplicity. From FRF measurements, LeBlanc et al. [1992] evaluated the damage effects on changes in the peaks of FRFs. The authors stated that, based on correct FRF analysis, FRF can accurately determine damage levels from the natural frequency and damping levels.

Many current methods for identifying structural damage, such as Genetic Algorithm (GA) and intelligent methods such as artificial neural networks (ANNs) are often implemented on the basis of some measured data and a large number of simulation data of structural vibration responses (Gomes et al. [2019b],Ribeiro Junior et al. [2020],Junior et al. [2021],Junior et al. [2022]). Therefore, Yan et al. [2006] emphasized that the establishment of a precise and efficient dynamic model for a structure is an important precondition. The authors presented an improved modeling method based on modifying the stiffness matrix of the element at the position of structural damage using a modifying coefficient. The influence of the position of structural damage and boundary conditions on the coefficient of modification for structural damage was verified, and for this the authors made use of FRF and natural frequencies. The stiffness matrix can be used in two different contexts. On the one hand, it is used in Finite Element Modelling (FEM) describing a part of the equation system that has to be solved. On the other hand, it describes material parameters in Hooke's law of elasticity where the relation between mechanical stress and strain is given (also known as stiffness tensor).

## 6 Final Remarks

In summary, this Chapter is focused on the discussion of the use of inverse problems as main strategy for parameter identification in structural systems. Basically, three direct answers can be used to formulate the inverse problem. Natural frequencies are more robust data that give global structural information and little local sensitivity. On the other hand, damping and mode shapes are responses with a greater amount of local information, however, more sensitive to noise. The quality of the inverse problem formulation also depends on the quality of the experimental test and also on the identification algorithms (gradient-based or gradient-free). Finally, inverse methods using modal data have a wide range of application and are under continuous development and improvement where new metrics and methodologies are developed.

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