

Chapter 1

Introduction to Probabilistic Methods & Uncertainty Modeling in Direct & Inverse Problems

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Introduction to Probabilistic Methods & Uncertainty Modeling in Direct & Inverse Problems

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Abstract

This chapter presents an overview of the Book Series in Direct Methods, Inverse Methods and Uncertainty Modeling, of which the first two volumes have already been published (Volume I: Model-Based and Signal-Based Inverse Methods, DOI: <https://doi.org/10.4322/978-65-86503-71-5>, and Volume II: Fundamental Concepts and Models for the Direct Problem, DOI: <https://doi.org/10.4322/978-65-86503-83-8>), and a presentation of the current Volume III: Uncertainty Modeling: Fundamental Concepts and Models. The chapter includes an introduction to the different topics in Optimization and Identification Techniques detailed in Volume I, an introduction to topics related to materials, technologies, and discrete modeling detailed in Volume II, and an introduction to topics related to uncertainty modeling and probabilistic methods, comprising the several chapters included in this Volume III of the Book Series.

1 Book Series in Discrete Models, Inverse Methods, & Uncertainty Modeling in Structural Integrity: overview

The Book series in “Discrete Models, Inverse Methods, & Uncertainty Modeling in Structural Integrity” is an initiative of the Post-Graduate Program - Integrity of Engineering Materials (PPG-Integridade) of University of Brasilia (UnB), organized as a collaborative work involving researchers, engineers, scholars, from several institutions, universities, industry, recognized both nationally and internationally.

This book series is an activity related to the Research, Development & Innovation (R,D&I) Project at UnB, titled: “Technological Demonstration Platform for Inverse Methods and Uncertainty Modeling Integrity of Structures and Components”, available at the UnB Central Library (in Portuguese) (Jorge, 2020).

The Book Series project is comprised by three Volumes:

- Volume I – Model-based and Signal-Based Inverse Methods (Jorge, Anflor, et al., 2022d)

- Volume II – Fundamental Concepts and Models for the Direct Problem (Jorge, Anflor, et al., 2022a)
 - o Part I – Technologies and Materials Modeling
 - o Part II - Discrete Modeling
- Volume III – Uncertainty Modeling: Fundamental Concepts and Models

The different book chapters were elaborated encompassing the relevant project topics, including chapters covering:

- Fundamentals, including topics such as: basic principles, concepts & foundations, for the Direct & Inverse Problems (including model-based and signal-based inverse methods), and for the Modeling of Uncertainties;
- Special Topics, Applications, and Research Review, including topics such as: research review, state-of-the-art & future trend topics, for the Direct & Inverse Problems (including model-based and signal-based inverse methods), and for the Modeling of Uncertainties.

The book chapters were prepared as a collaborative work by researchers, engineers, scholars, involved in research, development and applications in the related areas.

The research areas of interest throughout the book chapters include:

- Modeling of the inverse problem, monitoring & diagnosis / prognosis: models and methods for inverse problems, optimization methods (including techniques such as; multi-objective optimization, topology optimization, evolutionary optimization), Wavelets, Kalman Filter (KF), Particle Filter (PF), Machine Learning (ML), Artificial Intelligence (AI), Data Science (DS), for applications such as Structural Health Monitoring (SHM) (including impedance-based and Lamb Wave-based techniques), Health & Usage Monitoring Systems (HUMS);
- Modeling of the direct problem: mechanics of materials (including metallic materials, composites), structures (including civil, mechanical naval, aeronautical structures) machinery design and mechanical components, fracture mechanics, impact, fatigue, damage tolerance, integrity, mechanical vibrations, dynamics of structures, computational mechanics, including mathematical methods and numerical methods for discrete modeling for continuum mechanics (such as Finite Element Methods (FEM), Boundary Element Methods (BEM), Mesh-Free Methods (MFM));
- Probabilistic methods and modeling of uncertainties: probabilistic methods in engineering, Design of Experiments (DOE), Response Surface Methods (RSM), Risk & Reliability (including structural and system reliability), Uncertainty Modeling (UM) & Uncertainty Quantification (UQ), Bayesian Approaches (BA), stochastic Finite Element approaches (Stochastic FEM, Spectral FEM, Polynomial Chaos), stochastic optimization, meta-modeling (including techniques such as Surrogate Models (SM), Reduced Order Models (ROM)), model Verification & Validation (V&V).

The different models, methods and approaches presented throughout the several chapters in the three Volumes of this Book Series are intended as an introductory presentation of some possibilities of methods that could be used in problems related to integrity of structures and components, and maybe even extended to other engineering areas, as appropriate. The list of models is not unique, and is neither comprehensive nor

exhaustive, and the reader is encouraged to look for different possibilities of methods that may be applicable to the particular engineering problem at hand.

A common aphorism, often presented as "*All models are wrong, but some are useful*", is usually considered to be applicable to scientific models in general, and to statistical models in particular. The aphorism recognizes that statistical or scientific models always fall short of the complexities of reality but can still be of use. The aphorism is generally attributed to the statistician George E. P. Box, although the underlying concept predates Box's writings.

Section 2 presents an overview of Volume I - Model-Based and Signal-Based Inverse Methods, which includes chapters on several Optimization Models and Identification Techniques. An introduction and context is presented, as well as a listing of the chapters included in Volume I.

Section 3 presents an overview of Volume II - Fundamental Concepts and Models for the Direct Problem, which includes chapters distributed in Part I – Technologies and Materials Modeling, and in Part II - Discrete Modeling. An introduction and context is presented, as well as a listing of the chapters included in Volume II.

Section 4 presents an overview of Volume III - Uncertainty Modeling: Fundamental Concepts and Models, which includes chapters on Probabilistic Methods and Uncertainty Modeling. An introduction and context is presented, as well as a listing of the chapters included in Volume III.

Section 5 presents some final remarks and acknowledgements.

2 Overview of Volume I

2.1 Model-Based and Signal-Based Inverse Methods – context

The detection, localization, classification and identification of parameters and/or material properties, related to the integrity of structures and components, with and without defects or damages, involves the modeling of inverse problems, as well as an adequate modeling and quantification of the uncertainties involved in the problem.

The formulation of the direct problem, of the inverse problem, and the related uncertainties modeling, needed for an adequate description of the structure and/or the mechanical component, and of its potential defects or damages, involves multidisciplinary modeling techniques, whose understanding and proper application transcends the field of integrity and damage tolerance, being able to serve as a basis for applications, in other contexts or fields.

Among the application problems of interest for inverse methods, one can cite Structural health monitoring (SHM) and Health and Usage Monitoring Systems (HUMS).

The monitoring of structural integrity (SHM) is a competitive technique for damage detection and identification, wherein information is collected online, and compared with an existing database for an undamaged ("healthy") structure. From this comparison, real-time information on the presence of damages can be obtained, including their localization, size, propagation speed, and, ultimately, the remaining operational life of the structural component.

The monitoring of mechanical components (HUMS) is a technique which is being used to follow / accompany the state of the integrity of mechanical systems / components (Health) and to monitor the appearance of indicators of the presence of damage (usage) in dynamic systems, such as rotating components (in engines, for example) and in gearboxes (in mechanical transmission systems, for example). In this case, the comparison is made between vibration signals from the healthy components (accumulated historical data) and from the components being monitored, to identify significant discrepancies in the vibration signals, which could be correlated to specific / particular / known damages.

The scientific challenge of the modeling of inverse problems, as well as of the adequate modeling and quantification of the related uncertainties, in a problem of integrity of structures and components, involves several aspects:

- The modeling of the problems (direct problem, inverse problem, uncertainties) needs to be done, whenever possible, by using more than one technique, for each case being described, in order to implement, for the particular problem at hand, model techniques which are independent, complementary, and/or redundant. Whenever possible, more than one model should be used, for redundancy and/or comparative analysis, especially in the case of unavailability of prior data for the healthy structure and/or component.
- The techniques used for inverse methods may involve:
 - i. Optimization techniques, based on multi-objective optimization models, using classical optimization techniques (such as Sequential Quadratic Programming (SQP), BFGS, etc), or evolutionary optimization techniques (such as Genetic Algorithms (GA), Differential Evolution (DE), Sunflower Optimization (SFO), Lichtenberg Algorithm (LA), etc.);
 - ii. Identification techniques based on Artificial Intelligence, Machine Learning, Pattern Recognition, Data Science, etc., models (such as identification models based on Artificial Neural Networks (ANN), for example;
 - iii. Models based on the Wavelet Transform (continuous Wavelet Transform (CWT), discrete Wavelet transform (DWT), with different types and sizes of the Wavelet window, for example);
 - iv. Stochastic models (such as Kalman Filter (KF), Extended Kalman Filter (EKF), Extended Information Filter (EIF), Particle Filter (PF), Least Squares (LS), etc).
- In several situations, the direct models to be implemented may involve different problem physics (Multi-physics Modeling), and multiple scales (Multi-scale Modeling). In such cases, the description for the direct problem may involve coarser global models, and more detailed local models;
- The computational simulations and the experimental / laboratory tests must take into account the additional challenge of properly simulating / representing the local behavior of a complex structure, in the regions of interest, where the defect of damage is expected to be, or is expected to appear. For example, Fracture Mechanics (FM) problems and Damage Tolerance (DT) problems cannot be properly represented by reduced-scale models, as the damaged region must be represented using full-scale models. In these cases, the computational simulations (and also the experimental / laboratory tests) are required to reproduce the situation in the region of the damage using high fidelity local models. Thus, the region of the damage must be modeled in full scale, with the model also representing properly the

geometry, the mechanical properties, and the real loading in that local region (loading that is coming from the external loads that were applied in the structure or in the component as a whole).

- In many cases, inverse problems may belong to the category of ill-posed problems, which represents an additional challenge in the modeling of the problem at hand. In these cases, the approach for the inverse method may require additional hypothesis to be made (for problem regularization, for example), or that a meta-modeling approach is adopted (such as surrogate models, reduced order models, etc), replacing the original model by the proper meta-model, and then solving this approximate model for the problem.
- The modeling of inverse problems, such as in the case of SHM and/or HUMS, must take into account the proper modeling of the sensor behavior, and also the uncertainties associated to these sensors, as well as the simultaneous use of multiple, independent, techniques for monitoring, with different sensors. The optimal sensor positioning, to maximize the Probability of Detection (PoD), may be seen as a topological optimization problem and/or as a stochastic optimization problem.
- The modeling of inverse problems may involve the detection, localization, and identification of parameters and/or material properties (for example, properties such as elasticity modulus, Poisson's ratio, etc), which may vary through time (for example, material degradation through time) and also along the length of the structure or mechanical component (for example, local changes which may occur in the material properties and / or mechanical properties of a composite plate, due to the debonding between the layers of the composite material).

2.2 Chapter topics in Volume I: presentation

Along Volume I of the Book Series, several topics related to model-based and signal-based methods for inverse problems were presented in the several book chapters, representing the collaborative work from researchers, engineers, scholars, engaged in research, development and applications in the related areas, affiliated to several institutions, universities, industry, and recognized both nationally and internationally.

The book chapters in Volume I of the Book Series were distributed as follows:

- Chapter 1: Introduction to Optimization and Identification Techniques for Model-Based and Signal-Based Inverse Problems (Jorge, Anflor, et al., 2022b)
- Chapter 2: Overview of Some Optimization and Identification Techniques for Inverse Problems of Detection, Localization and Parameter Estimation (Sousa et al., 2022)
- Chapter 3: An overview of Linear and Non-linear Programming methods for Structural Optimization (Choze, Santos, Jorge, et al., 2022)
- Chapter 4: Overview of Traditional and Recent Heuristic Optimization Methods (Choze, Santos, & Gomes, 2022)
- Chapter 5: Application of Machine Learning and Multi-Disciplinary/Multi-Objective Optimization Techniques for Conceptual Aircraft Design (Mattos et al., 2022)
- Chapter 6: On a Bio-Inspired Method for Topology Optimization via Map L-Systems and Fractone Modeling (Kobayashi, 2022)

- Chapter 7: Fundamentals on the Topological Derivative concept and its classical applications (Carvalho et al., 2022)
- Chapter 8: Ultrasound Obstacle Identification using the Boundary Element and Topological Derivative Methods (Cisilino & Anflor, 2022)
- Chapter 9: Fundamental Concepts on Wavelet Transforms (Palechor, Silva, Morais, et al., 2022)
- Chapter 10: Application of Wavelet Transforms to Structural Damage Monitoring and Detection (Palechor, Silva, Gomes, et al., 2022)
- Chapter 11: Inverse Methods using KF, EKF, EIF, PF, and LS Techniques for Detection, Localization, and Parameter Estimation (Myers & Jorge, 2022)
- Chapter 12: Fundamental Concepts for Impedance-based Structural Health Monitoring (Finzi Neto & Moura, 2022)
- Chapter 13: Fundamental Concepts for Guided Lamb Wave-based Structural Health Monitoring (Finzi Neto et al., 2022)
- Chapter 14: Machine Learning and Pattern Recognition: Methods and Applications for Integrity Monitoring of Civil Engineering Structures (Alves et al., 2022)

3 Overview of Volume II

3.1 Fundamental Concepts and Models for the Direct Problem – context

The modeling of the Direct Problem includes disciplines related to:

- Materials Science, Mechanics of Materials (applications to engineering materials, such as metallic materials, composites, biomaterials, etc.), Material Characterization Techniques (Digital Image Correlation (DIC), etc.);
- Modeling & Design of Structures (applications to several Engineering branches such as Mechanical, Aeronautical, Civil, Naval, Ocean & Offshore, Nuclear, Energy, Oil & Gas, etc.);
- Design of Machines and Mechanical Components;
- Manufacturing Technologies and Processes (Additive Manufacturing, etc.)
- Structural & Component Integrity (Fracture Mechanics, Fatigue, Impact, Contact, Damage Tolerance, Inspection, Non-Destructive Testing (NDE), Maintenance, Structural Health Monitoring (SHM), Health and Usage Monitoring Systems (HUMS), etc.);
- Mechanical Vibrations, Acoustics, Structural Dynamics (Data Acquisition, Modal Testing & Modal Analysis Techniques, etc.);
- Computational Mechanics, Mathematical Methods, Numerical Methods (such as Finite Element Methods (FEM), Boundary Element Methods (BEM), Meshless / Meshfree Methods, etc.).

Regarding structural integrity applications, the problem of detecting damage in a structure involves modeling the direct problem, that is, the behavior of that structure in the presence of one or more pre-established damages, of assumed shape and size, and at given locations. This modeling can be done, for example, involving the analysis of the distribution of stresses and strains in parts with cracks, done through a FEM, BEM or Meshless modeling, or by analyzing the distribution of sound waves in this cracked part, emitted by a pre-established source, by means of a FEM, BEM or Meshless modeling.

Regarding component integrity applications, for the problem of detecting faults in dynamic components, experimental vibration data can be acquired from accelerometers conveniently distributed in the vicinity of the components of interest. In this case, there would be no direct problem modeling (the direct model is implicit), as the vibration data would be acquired for the healthy components to be later compared with the vibration data of the defective (with failures) components.

With respect to Direct Problem Modeling, several problems of interest in structural integrity can be modeled as initial value problems and/or boundary value problems. In the case of boundary value problems, not varying in time, there is a valid partial differential equation in the region that represents the problem domain, and valid boundary conditions at the boundary of this region. In the case of time-dependent initial value problems, there are also valid initial conditions in this domain.

A boundary value problem can be represented by a general equation of type $Au = p$, valid in the domain D , with boundary conditions valid in the boundary ∂D . The complexity of the problem, the geometry of the boundary, the actions (external loads, restrictions on supports) limit the possibilities of analytical solution for this problem, which generates a demand for approximate solutions, such as, for example, through:

- replacement of the differential operator A by an approximate operator A^* . This is the case of the finite difference method, which adopts an algebraic operator A^* , where the derivative at a given point is expressed through the values of the function at points located in a neighborhood of that point, allowing the construction of a system of linear equations, to be solved with the proper application of the initial and boundary conditions;
- replacement of the variable u by an approximate variable u^* , expressed in the form of a linear combination of a set of basic functions. These functions can be polynomial, trigonometric, etc. The coefficients of this linear combination are the components of this approximate solution in the coordinate system represented by these basic functions. When the number of components tends to infinity, the approximate solution u^* tends to the solution u . Among the approximate solutions based on this method, many correspond to methods called “projective”, where the equation valid in the domain is projected in an auxiliary subspace of functions, called support functions, and integrated in the domain, with its boundary conditions, using integration by parts. This category of methods includes, among others:
 - o The Finite Element Method (FEM), using an integration by parts and, in each element, the same functions, both for support and for approximation of the variable u , in order to obtain the solution in points of the domain, called nodes, and, by interpolation using these functions, the solution at all other points in the domain;
 - o The Boundary Element Method (BEM), using two integrations by parts and using support functions called fundamental solutions, in order to eliminate the domain integral, leaving only the boundary integrals (also known as boundary integrals). Thus, the solution is initially obtained only at the nodes of the discretized contour. Then, by interpolation, the solution is found at the other points of the contour. Finally, by post-processing, the solution is obtained at any desired point in the domain.

For problems of interest in structural integrity, the finite element method can be efficiently applied to problems in which the domain is inhomogeneous, where the discretization of the elements would already consider this local non-homogeneity.

On the other hand, problems involving domain singularities, such as stress distribution in the vicinity of the crack tip, in fracture mechanics, can be well represented using the boundary element method.

For initial value problems, with variable (in the domain and/or in the frontier) dependent on time, it may need a combination of methods that include the finite difference method, to model the march in time of the variable of interest.

Other numerical methods are also available, such as the Meshless / Meshfree Methods, in which the global system of equilibrium equations is constructed using a node-by-node process, performed in the local domain of each node, and thus there is no need to define *a priori* a mesh of finite elements or a mesh of boundary elements.

For the application of inverse methods in dynamic components, the methodology is based on the comparison of vibration data acquired and stored throughout the life of the components still undamaged, with the most recent vibration data, for the components currently in use, from so that significant changes in the spectrum of acquired signals can be used as a starting point for the detection and identification of damage to these components. In this case, instead of a numerical code for the direct problem, vibration and acoustics data acquisition systems are used, together with structural dynamics, vibration analysis, modal testing & modal analysis techniques.

The direct problem modeling can also include the study of the damage evolution with time (such as, for example, the crack propagation rate), in order to estimate the remaining useful life (safe life) of the structure.

The science and technology behind analyzing structures is not yet mature, and further advancements and research are unnecessary for the basic understanding of structural mechanics phenomena and behavior (design/analysis/test) (Hamm et al., 2021). Some remaining challenges in the structures discipline include many areas still in need of rigorous research to arrive at physics-based methodologies, including: Fracture Mechanics, Composite Materials, New Materials, Manufacturing and Processing Methods, New Applications, Lack of Physics-Based Theories, New Test Methods, Incorporation of Research Advancements, Computational Advances, Multi-Discipline Interactions.

One final note; in the context of philosophy of science, two fundamental issues were pointed out in (Da Costa & French, 2006). One issue concerns the appropriate attitude we should take towards scientific theories - whether we should regard them as true or merely empirically adequate, for example. The other issue concerns the nature of scientific theories and models and how these might best be represented. These two issues together lead us to arguing that theories and models should be regarded as partially rather than wholly true, with issues in terms of belief, theory acceptance, and the realism-antirealism debate.

3.2 Chapter topics in Volume II: presentation

Along Volume II of the Book Series, several topics related to technologies, materials and discrete modeling for direct problems are presented in the several book chapters, representing the collaborative work from researchers, engineers, scholars, engaged in research, development and applications in the related areas, affiliated to several institutions, universities, industry, and recognized both nationally and internationally.

The book chapters in Volume II of the Book Series are distributed as follows:

- Chapter 1: Introduction to the Direct Problem: Materials, Technologies, and Discrete Modeling (Jorge, Anflor, et al., 2022c)

Part I - Technologies, Structural Integrity & Material Modeling

- Chapter 2: Technologies and Materials Issues Related to Naval and Ocean Engineering (Marques, 2022)
- Chapter 3: New Advances in Thermoplastic Commingled Composites: Processing and Molecular Dynamics (Benedetto et al., 2022)
- Chapter 4: The Influence of Weaving Patterns on the Effective Mechanical Response of Reinforced Composites - A Study Through Homogenization (Rocha et al., 2022)
- Chapter 5: Auxetic Structures: Parametric Optimization, Additive Manufacturing, and Applications (Francisco et al., 2022)
- Chapter 6: Additive Manufacturing of Ti-Alloys for Aerospace, Naval and Offshore Applications (Mendoza et al., 2022)
- Chapter 7: Review of Low Cycle Fatigue: Issues in Naval and Offshore Engineering (Delgado-Morales et al., 2022)
- Chapter 8: Modeling of Damage in Turbine Blades for Large Deformations (Aslan, 2022)
- Chapter 9: Stresses on Expansion Joints Used in Penetrations (Suanno & Maneschy, 2022)
- Chapter 10: Leak-Before-Break Technology for Piping in Nuclear Industry (Maneschy, 2022)
- Chapter 11: Fatigue Control in Angra Nuclear Power Plants (Cisternas, 2022)
- Chapter 12: On the use of Modal Test Data in Inverse Problems: Fundamentals and Applications (Gomes et al., 2022)
- Chapter 13: Application of Deep Learning Techniques for the Impedance-based SHM to the Oil & Gas Industry (Rezende et al., 2022)
- Chapter 14: Noise, Vibration, and Health and Usage Monitoring Systems (HUMS) of Aircraft Dynamic Components (Cruz et al., 2022)
- Chapter 15: On Vibration Analysis and Health and Usage Monitoring Systems (HUMS) of Dynamic Components of Helicopters (Jorge et al., 2022)
- Chapter 16: Model-Based Parameter Identification for Helicopter Main Rotor Balancing and Tracking Using Once-per-Revolution Vibration Data (Jorge et al., 2022)

Part II - Discrete Modeling: Finite Element, Boundary Element, Meshfree Methods

- Chapter 17: Finite Element Method for Structural Integrity Problems (Sales et al., 2022)
- Chapter 18: An Application of Finite Element Method and Sensitivity Analysis in Structural Dynamics (Avila & Prado, 2022)
- Chapter 19: The Boundary Element Method for Potential Problems (Albuquerque, Jorge, et al., 2022)
- Chapter 20: The Boundary Element Method for Structural Problems (Albuquerque, Sollero, et al., 2022)
- Chapter 21: The Boundary Element Method for Damage Modeling (Peixoto & Chaves, 2022)
- Chapter 22: Application of Roving Mass Technique Associated with Wavelet Transform to Structural Damage Detection and Localization (Palechor, Santos, et al., 2022)
- Chapter 23: Local Mesh Free Methods in Linear Elasticity and Fracture Mechanics (Oliveira et al., 2022)
- Chapter 24: Meshless smoothed point interpolation methods for damage modeling (Gori, 2022)
- Chapter 25: A New Non-Iterative Reconstruction Method for Solving a Class of Inverse Problems (Novotny, 2022)

4 Overview of Volume III

4.1 Uncertainty Modeling: Fundamental Concepts and Models – context

The modeling and quantification of uncertainties includes disciplines related to:

- the stochasticity of the problem variables, which may be modeled as:
 - random variables (univariate or multivariate, Gaussian Mixture Models, etc.) with parameters (for example: mean; standard deviation) constant in space and time (stationary processes); or
 - stochastic processes (when these parameters are variable over time), described by time series; or
 - random fields (when there is a spatial distribution of the parameters of the different random variables along the structure or the mechanical component). In stochastic finite element methods (SFEM), the discretization of the random fields (related to the distributed material properties, or to the distributed loading, for example) may be different than the finite element discretization itself (in general, one element of the random field may be related to a greater number of finite elements);
- probabilistic methods in engineering, including techniques such as Design of Experiments (DoE), Response Surface Methodology (RSM), Bayesian Approaches, Metamodels (Surrogate Models, Reduced Order Models), sensitivity analysis, stochastic optimization (robust optimization / multi-objective optimization) techniques, reliability methods, etc.

- the replacement of the real problem with a model to be simulated and / or tested, leading to several types of errors, such as: model errors (lack of fidelity of the model to the original problem); random errors (improper or incorrect account of the randomness of the problem variables); discretization errors (use of coarse meshes versus mesh refinement), numerical errors (such as rounding errors, etc.). The modeling and quantification of uncertainties may require the use of concepts related to Model Validation & Verification.

4.2 Chapter topics in Volume III: presentation

Along this Volume III of the Book Series, several topics related to uncertainty modeling and probabilistic methods for the direct and inverse problems are presented in the several book chapters, representing the collaborative work from researchers, engineers, scholars, engaged in research, development and applications in the related areas, affiliated to several institutions, universities, industry, and recognized both nationally and internationally.

The book chapters in this Volume III of the Book Series are distributed as follows:

- Chapter 1: Introduction to Probabilistic Methods & Uncertainty Modeling in Direct & Inverse Problems
- Chapter 2: Application of Data Science in the Sustainable Energy Industry
- Chapter 3: Overview of Classic Surrogates and Metamodeling Strategies in Structural Analysis
- Chapter 4: Uncertainty Quantification in Inverse Kinematics Computation of Space Robots by Neural Networks
- Chapter 5: Metamodel-Based Simulation Optimization: A Systematic Literature Review
- Chapter 6: Principal Component Analysis: An Overview and Applications in Multivariate Engineering Problems
- Chapter 7: Gaussian Mixture Models and Their Applications to Fault Diagnosis in Electric Motors
- Chapter 8: Sensor Placement Optimization of a Helicopter Main Rotor Blade
- Chapter 9: Challenges for Structural Health Monitoring: Nonlinearities and Uncertainties
- Chapter 10: Applications of Numerical and Statistical Methods for Monitoring and Control Engineering Variables in a Concrete Dam
- Chapter 11: Application of Statistical Processing Techniques to the Impedance-based SHM for the Oil & Gas Industry
- Chapter 12: Neuro-Fuzzy Data Normalization Applied for Impedance-based SHM to the Oil & Gas Industry
- Chapter 13: On Kriging Techniques & Impedance-based SHM as Applied to Damage Detection in 2D Structures
- Chapter 14: Time Series Models and Expansion Optimal Linear Estimation for Time-Dependent Reliability Analysis
- Chapter 15: Bayesian Optimal Experimental Design

- Chapter 16: Airplane Optimal Robust Design Combined with an Entropy-Statistics Unsupervised Learning Classification Algorithm
- Chapter 17: Random Fatigue Under Uniaxial and Multiaxial Loading Conditions
- Chapter 18: Random Field Discretization in Stochastic Finite Elements: Element Size Effects and Error Indicators in Reliability Structural Analysis
- Chapter 19: On Error Estimators and Bayesian Approaches in Computational Model Validation

5 Final remarks and acknowledgements

This Chapter presents an overview of the Book Series in Direct Methods, Inverse Methods and Uncertainty Modeling, with focus on:

- The previously published Volume I: Model-Based and Signal-Based Inverse Methods (DOI: <https://doi.org/10.4322/978-65-86503-71-5>), including an introduction to the different topics in Optimization and Identification Techniques detailed in Volume I of the Book Series;
- The previously published Volume II: Fundamental Concepts and Models for the Direct Problem (DOI: <https://doi.org/10.4322/978-65-86503-83-8>), including an introduction to the topics in Materials, Technologies and Discrete Models, detailed in Volume II of the Book Series;
- The current Volume III: Uncertainty Modeling: Fundamental Concepts and Models, including an introduction to the topics in Uncertainty Modeling and Probabilistic Methods, presented in this Volume III of the Book Series.

This Book Series is an initiative of the Graduate Program in Integrity of Engineering Materials (PPG-Integridade) at the University of Brasilia (UnB), Brazil (www.pgintegridade.unb.br). The editors would like to thank PPG-Integridade and UnB for the initiative, incentive and support for this Book Series project.

The book series is organized as a collaborative work involving researchers, engineers, scholars, engaged in research, development and applications in the related areas, affiliated to several institutions, universities, industry, and recognized both nationally and internationally. The editors are grateful and would like to show their appreciation to all the co-authors of the book chapters, for their participation, dedication, and support.

The book series is published as a digital version, with ISBN provided by UnB, and DOI for each chapter, provided by the Latin American Journal of Solids and Structures (LAJSS) (www.lajss.org).

The scope of the Book series is in the broad areas of interest of LAJSS, and also of the Brazilian Society of Mechanical Sciences and Engineering (ABCM) (www.abcm.org.br) and the Brazilian Association of Computational Methods in Engineering (ABMEC) (www.abmec.org.br). For increased visibility, these three institutions are encouraging the divulgation of the Book Series project in their websites. The editors would like to express their appreciation to LAJSS, ABCM and ABMEC, for their incentive, encouragement and support for this Book Series project.

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