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On-Line Process Control of the Number of Non-Conformities in the Inspected Item

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Abstract

Generally, production systems as automatic welding process, production of ceramic products, making clothes use automatic control and to evaluate the quality of their production processes, they employ on-line process control. The control system consists of a periodic inspection of one item after every m produced items. The number of non-conformities is monitored in the inspected item and if it exceeds the control limit, then it is decided that the process is out-of-control and the process is stopped for adjustment, otherwise the production continues. The process starts in-control with a fixed non-conformities rate and, after an assignable cause, this rate increases leading the system to operate out of control. The process remains in these conditions until the change is detected and the process adjusted. After adjustment, the process returns to operate in-control. The aim of this paper is to present an economic approach to monitor the rate of non-conformities in a production by on-line process control. To design such type of process, an average cost per item produced is achieved through the properties of an ergodic Markov chain and the two required parameters: the inspection interval and the upper control limit are obtained by minimizing the average cost per produced item. A numerical example illustrates the proposal. It was identified the most important factors which result a considerable impact on the average cost per item: the probability of a shift in the parameter of Poisson distribution; cost to send non-conforming items to the customers; the in-control non-conformity rate; the specification limit and the cost of adjustment.

Keywords: *On-line process control, Rate of non-conformities, Markov chain, Economic model, Poisson distribution.*

Introduction

Many of the articles related to on-line process control by attributes consist to inspect an item at every m produced ones and judge it as conforming or nonconforming. If the inspected item is classified as conforming the production continues (it is said the production system is operating in-control), otherwise the production process is adjusted and the production is restarted.

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This approach is economically feasible since the average cost per item produced is minimized to determinate the optimum parameters of the proposed procedure. However, in many practical situations it does not make sense classifying the item inspected only as conforming or nonconforming. The inspector may identify a small defect or nonconformity such as a fridge drawer without a latch or a scratch. In this case, the refrigerator may work perfectly but it only presents a single nonconformity. In some cases, it is more interesting to count the number of non-conformities in the inspected item (for example, the refrigerator) than simply classify it as non-conforming or conforming. In this context, many articles that present economic procedures for on-line quality control by attributes can be found in literature, however it is not common to find papers that consider the number of nonconformities in the inspected unit as the monitored statistic.

In general, production systems using automatic control like automatic welding process, production of ceramic products, semiconductor production, making clothes, manufacture of rings and bracelets, production of diodes used in printed circuit boards and chemical processes can benefit from the methodology discussed here.

Pioneering works as Taguchi (1981) and Taguchi *et al*. (1989) proposed a procedure for on-line control of process for variables and attributes where it was assumed that the process begins in state I (in-control) and after a special cause, the process starts to operate in the state II (out of control). The production remains in this condition until the changing is detected and the special cause removed. In the proposed control system one item is inspected at every m items produced and if the inspected item does not meet an established criterion, it is assumed that the process is out of control and the process is stopped for adjustment. After adjustment, the process is restarted in-control again. This type of control is suitable for process with a high production volume. In Taguchi (1981) and Taguchi *et al*. (1989) the fraction of conformance is used to on-line control by attributes. In this case, if the inspected item is conforming the process is classified as in-control. Analytical expressions are obtained to calculate the sampling interval (m) that minimizes the average cost of control system. However, they did not assume an explicit mechanism for the occurrence of the special cause as a series of simplifications and unrealistic situations are used to obtain such analytical expressions. Due to this Taguchi (1981) and Taguchi *et al*. (1989) received considerable criticism. Among various contributions about on-line process control for attributes, the papers of Nayebpour and Woodall (1993) and Nandi and Shreehari (1997, 1999) can be cited. In the first article, the authors assume that the time that the process remains in-control follows a geometric distribution of parameter π. Nandi and Shreehari (1997) presented a model considering two special causes and Nandi and Shreehari (1999) use a continuous function of deterioration in the quality of the production process. In both papers, no misclassification in the inspected item is assumed. However, in practical situations, the classification errors exist and should not be ignored, that is, a conforming item is wrongly classified as nonconforming or a nonconforming item classified as conforming. For more details about inspection errors, see (Johnson *et al*., 1991). Borges *et al*. (2001) evaluated the impact of classification errors in on-line process control for attributes. Even small classification errors less than 1% lead an increase in the inspection interval and, generally, a process with misclassification yields higher average cost. To reduce the impact of classification errors in the average cost, one may suggestion the repeated classifications. That is, the item is inspected r times independently and classified as

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conforming or not according to some criteria. Further details see Quinino and Suyama (2002), Quinino and Ho (2004) and Trindade *et al*. (2007a, b). A hybrid model based on the sequential results of repetitive classifications was proposed by Quinino and Ho (2004) and Quinino *et al*. (2010).

This article aims to propose on-line process control to monitor the rate of nonconformity. For that a probabilistic model is developed considering the number of non-conformities of the inspected item follows Poisson distribution with an average rate of defects as the parameter to be monitored. The model considers an inspection system which may be built by a set of discrete states of a to determine an optimal strategy of control. Here a long-run production will be considered. The optimal strategy is to minimize the average cost of the control system (per item) to determined the sampling interval (m) and the upper control limit (L). The paper is organized as follows: the probabilistic model is presented in section 2. In the section 3 the average cost of the control system is obtained and a numerical example with sensitivity analysis is presented in section 4 to illustrate the proposed procedure. The conclusions and suggestions are included in the section 5.

Probabilistic Model

Let us consider a continuous system of control. The process is said to bein-control if the items are produced with a rate of non-conformity λ_0 (state I). On the other hand, when the items are produced at rate of non-conformity λ_1 (state II, $0 \leq \lambda_0 < \lambda_1$), it is said process is out of control. The shift from the state I to state II is described by a geometric distribution with parameter π, $0 \leq \pi \leq 1$. It is assumed that C, the number of nonconformities in the inspected item follows Poisson distribution with parameter λ. The control system consists of inspecting the m-th item after a cycle of m produced items. In each inspected item, if $C > L$, L, the upper control limit, the process is declared out of control and it is stopped for adjustment. It is assumed that the lower control limit is zero.

Like tests of hypothesis, the decision is subject to two types of errors: declare the process as out of control when it is in-control (α) and declaring the process in-control when it is out of control (β). If the process is judged out of control, it is assumed a shift in the rate of non-conformity and the production is interrupted for adjustment. When the process is adjusted, it is restarted in state I and the inspected item is discarded. At state II the process can only return to state I after an adjustment.

The inspection system can be modeled as a stationary Markov chain considering the set of discrete states E = {01, 00, 11, 10, 21, 20}. The first index (W) indicates the real state of the process when the items that make up the inspection cycle were produced. When W = 0, all items, including the inspected are produced in the state I. When W = 1, a shift from state I to state II occurred in the current cycle and at least the inspected item was produced in the state II. When W = 2, all items, including the inspected were produced in the state II. The second index (denoted by V) indicates whether the process was declared out of control (V = 0) and the production is stopped for adjustment or in-control (V = 1) and the production goes on. Figure 1 represents the flowchart of the control of production process.

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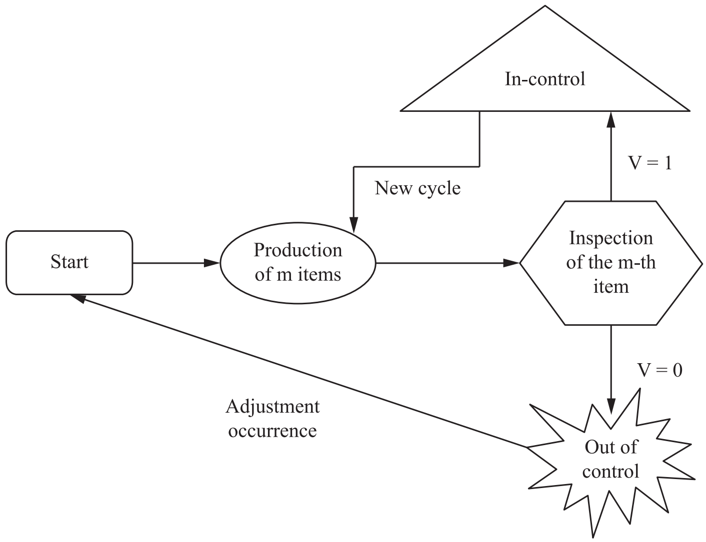

Figure 1. Flowchart of the production system.

Following the transition probabilities of the states of the Markov Chain are described. The notation $P_{(wv)(w^*v^*)}$, with w,w* = 0,1,2 and v,v* = 0,1 will be used here on. For example, $P_{(01)(01)}$, denotes P(E_{i+1} = *01* | E_i = *01*). That is, all items of the current cycle (i + 1) are produced at state I (w = 0) and no adjustment since the number of non-conformities of the inspected item is less than the upper control limit *L* (*v* = *1*) and the previous cycle (*i*) all items were also produced at the state I (*w* = *0*) and no adjustment (*v* = *1*) since the number of non-conformities of the inspected item was less than L. So

$$P_{(01)(01)} = P\{\Delta_m = \lambda_0\} \cdot P\left[C \le L \mid \lambda = \lambda_0\right] = (1-\pi)^m \cdot (1-\alpha) \tag{1}$$

With $P\{\Delta_m = \lambda_0\} = P\{\Delta_1 = \lambda_0, \Delta_2 = \lambda_0, \cdots, \Delta_m = \lambda_0\} = (1-\pi)^m$ the probability of all m items are produced in state I in a cycle; Δ_i, $i \geq 0$, denotes the state (a non-observable random variable) in which the i-th item was produced and $\alpha = P(C > L|\lambda = \lambda_0)$ is the probability of a process be wrongly judged as out of control. The probabilities $P_{(20)(01)}$, $P_{(00)(01)}$ and $P_{(10)(01)}$ indicate that in the previous cycle, the number of non-conformities in the inspected item does not meet the control limit L (the process is adjusted and it restarts in-control). Thus the following equalities are valid:

$$P_{(01)(01)} = P_{(00)(01)} = P_{(10)(01)} = P_{(20)(01)} \tag{2}$$

The probability $P_{(01)(00)}$, denotes P(E_{i+1} = *00* | E_i = *01*). That is, all items of the current cycle (*i* + *1*) are produced at state I (W = *0*) and it is wrongly decided that the process is out of control since the number of non-conformities of the inspected item is higher than the upper control limit *L* (V = *0*). At the previous cycle (*i*) all items were also produced at the state I (W = *0*) and no adjustment (V = *1*) since the number of non-conformities of the inspected item was less than L, so consequently:

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$$P_{(01)(00)} = P\{\Delta_m = \lambda_0\} \cdot P[C > L \mid \lambda = \lambda_0] = (1-\pi)^m \cdot \alpha \tag{3}$$

Similarly, the probabilities $P_{(00)(00)}$, $P_{(10)(00)}$, $P_{(20)(00)}$ mean that in the previous cycle, the process was judged as out of control, adjusted and it restarts in-control. So the next equalities hold

$$P_{(01)(00)} = P_{(00)(00)} = P_{(10)(00)} = P_{(20)(00)} \tag{4}$$

For $P_{(01)(11)}$ and $P_{(01)(10)}$the process was in-control and the inspected item met the control limits in the previous cycle. In the current cycle a shift of the rate non-conformity from λ_0 to λ_1 occurred (W = 1). So at the least the inspected one was produced at state II. The process is wrongly (correctly) decided that is in-control (out of control) at first (second) case. So,

$$P_{(01)(11)} = [1 - P\{\Delta_m = \lambda_0\}].P[C \le L \mid \lambda = \lambda_1] = [1-(1-\pi)^m].\beta$$

$$P_{(01)(10)} = [1 - P\{\Delta_m = \lambda_0\}].P[C > L \mid \lambda = \lambda_1] = [1-(1-\pi)^m].(1-\beta) \tag{5}$$

with $\beta = P(C \le L | \lambda = \lambda_1)$.

Similarly the next equalities follow:

$$P_{(01)(11)} = P_{(00)(11)} = P_{(10)(11)} = P_{(20)(11)}$$

$$P_{(01)(10)} = P_{(00)(10)} = P_{(10)(10)} = P_{(20)(10)} \tag{6}$$

In the probabilities $P_{(11)(21)}$ and $P_{(11)(20)}$ the parameter shifted in the previous cycle. So in the current cycle all items are produced at state II. The first (second) probability indicates the process is wrongly (correctly) judged as in-control (out of control).

$$P_{(11)(21)} = P[C \le L \mid \lambda = \lambda_1] = \beta$$

$$P_{(11)(20)} = P[C > L \mid \lambda = \lambda_1] = 1\text{-}\beta \tag{7}$$

And finally the transitory probabilities $P_{(21)(21)}$, $P_{(21)(20)}$ indicate that the parameter has shifted in previous cycle and still at this state in the current cycle, so

$$P_{(21)(21)} = P_{(11)(21)}$$

$$P_{(21)(20)} = P_{(11)(20)} \tag{8}$$

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Employing expressions (1)-(9) the **P** transition matrix in (9) can be expressed as

$$P = \begin{array}{c c} & \begin{array}{cccccc} 01 & 00 & 11 & 10 & 21 & 20 \end{array} \\ \begin{array}{c} 01 \\ 00 \\ 11 \\ 10 \\ 21 \\ 20 \end{array} & \begin{bmatrix} P_{(01)(01)} & P_{(01)(00)} & P_{(01)(11)} & P_{(01)(10)} & 0 & 0 \\ P_{(01)(01)} & P_{(01)(00)} & P_{(01)(11)} & P_{(01)(10)} & 0 & 0 \\ 0 & 0 & 0 & 0 & P_{(11)(21)} & P_{(11)(20)} \\ P_{(01)(01)} & P_{(01)(00)} & P_{(01)(11)} & P_{(01)(10)} & 0 & 0 \\ 0 & 0 & 0 & 0 & P_{(11)(21)} & P_{(11)(20)} \\ P_{(01)(01)} & P_{(01)(00)} & P_{(01)(11)} & P_{(01)(10)} & 0 & 0 \end{bmatrix} \end{array} \qquad (9)$$

The number of states of the transition matrix **P** is finite and all states are recurrent and aperiodic so this Markov chain is an ergodic one (see details in Ross, 2005) and then $lim_{j\to\infty} \boldsymbol{P}^{(j)} = \boldsymbol{Z}$ exists in which all rows of the matrix $\boldsymbol{Z}$ are equal to the row vector $\mathbf{z} = [z_{(01)}, z_{(00)}, ---, z_{(20)}]$. The vector **z** is a vector of probabilities $\left(\sum_i z_i = 1\right)$ in the stationary state with all z_i values strictly positive. The element $\boldsymbol{Z}_{(wv)}$, $w = 0, 1, 2$; $v = 0,1$ can be interpreted as the proportion of time that the process stays at state (wv) for a sufficiently large number of inspections. As $\boldsymbol{P}^{(j+1)} = \boldsymbol{P}^j \boldsymbol{P}$ and $lim_{j\to\infty} \boldsymbol{P}^{(j+1)} = lim_{j\to\infty} \boldsymbol{P}^{(j)} = \boldsymbol{Z}$ then it follows the equality $\boldsymbol{Z} = \boldsymbol{ZP}$. As all rows of $\boldsymbol{Z}$ are equals to $\boldsymbol{z}$, the equality $\boldsymbol{z} = \boldsymbol{zP}$ also holds. So it can be written as

$$\boldsymbol{z} = \boldsymbol{zP} \Rightarrow \boldsymbol{z}\left(\boldsymbol{P} - \boldsymbol{I}\right) = \boldsymbol{0} \qquad (10)$$

where **I** is the identity matrix and **0** the null vector. Therefore, the vector **z** can be obtained from solving the linear system (10) with the restriction that $\sum_{w,v} z_{(wv)} = 1$. Solving (10), the elements of **z** are given by

$$z_{(01)} = \frac{P_{(01)(01)} P_{(11)(20)}}{P_{(01)(11)} + P_{(11)(20)}} \quad z_{(00)} = \frac{P_{(01)(00)} P_{(11)(20)}}{P_{(01)(11)} + P_{(11)(20)}}$$

$$z_{(10)} = \frac{P_{(01)(10)} P_{(11)(20)}}{P_{(01)(11)} + P_{(11)(20)}} \quad z_{(21)} = \frac{P_{(01)(11)}\left(1 - P_{(11)(20)}\right)}{P_{(01)(11)} + P_{(11)(20)}}$$

$$z_{(11)} = z_{(20)} = \frac{P_{(01)(11)} P_{(11)(20)}}{P_{(01)(11)} + P_{(11)(20)}}$$

Average Cost of the Control System

In this section the average cost of the control system will be described. It follows the structure of economical designs. So the costs considered in the current study are:

- c_I - cost to inspect an item;

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- c_{nc} - cost to send a non-conforming to the customers or another stages of production. Note that an item is classified as conforming if the inspected one meets the upper specification limit LE (the lower is equal zero), that is $C \leq LE$;
- c_a - cost to adjust the process;
- c_{s_nc} - cost to scrap a non-conforming; and
- c_{s_c} - cost to scrap a conforming item.

The components c_{s_nc} and c_{s_c} are used if discarded items may be submitted to a process of retrieval. So costs may be different for a conforming or a non-conforming one. The cost of each state (wv) can be written as $T_{(wv)} = C_1 + \xi_{(wv)} + \eta_{(wv)} + \phi_{(wv)}$; $w = 0, 1, 2$ and $v = 0, 1$:

- $\xi_{(wv)}$, Cost to send a non-conforming item for the customer or to the later stages of the process;
- $\eta_{(wv)}$, Cost to scrap an item inspected;
- $\varphi_{(wv)}$, Cost of adjustment of the process.

Below the costs are detailed. Consider $p_1 = P(C \leq LE|\lambda = \lambda_0)$ and $p_2 = P\ (C \leq LE|\lambda = \lambda_1)$, the probabilities of the inspected item to be declared as conforming, respectively, when the process is in-control, and out of control. For the states (00) and (01), all items m were produced at the state I in the current cycle. The expected number of the non-conforming items is $(m - 1)\ [1 - P\ (C \leq LE|\lambda = \lambda_0)]$ among the $(m - 1)$ items sent to the customer. Thus, the cost to send non-conforming items to the customer or the later stages is

$$\xi(00) = \xi(01) = c_{nc}(m - 1)\ [1 - p_1]$$

Similarly for states (20) and (21), all items in the current cycle are produced at the state II. Thus the cost of sending non-conforming items for the consumer or the later stages is:

$$\xi(20) = \xi(21) = c_{nc}(m - 1)\ [1 - p_2]$$

For states (11) and (10), i items in m are produced at state I and the others $(m - i)$ produced at state II. Thus the cost of sending non-conforming items for the customer or the later stages considering all possibilities is:

$$\xi(11) = \xi(10) = c_{nc} \sum_{i=1}^{m} \frac{\pi(1-\pi)^{i-1}}{1-(1-\pi)^m} \left\{(i-1)[1-p_1] + (m-i)[1-p_2]\right\}$$

About the cost to scrap the inspected item, at state (00) all items m are produced at state I and the process was declared out of control. Thus, the cost of scrapping the item inspected is:

$$\eta(00) = c_{s\_c} \frac{P(L < C < LE)}{\alpha} + c_{s\_nc} \frac{P(C > \max(L, LE))}{\alpha}$$

At state (01), the process is declared in-control, but the inspected item can be conforming or non-conforming, so the cost to scrap it is

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$$\eta(01) = c_{s\_nc}\frac{P(LE < C < L)}{1-\alpha} + c_{s\_c}\frac{P(C < \min(L, LE))}{1-\alpha}$$

At state (*20*), all items were produced at state II while at state (*10*) at least an inspected item was produced at state II. In both cases, the process was considered out of control, so

$$\eta(20) = \eta(10) = c_{s\_c}\frac{P(L < C < LE)}{1-\beta} + c_{s\_nc}\frac{P(C > \max(L, LE))}{1-\beta}$$

But at states (*21*) and (*11*), the process was wrongly considered in-control. Hence, the costs to scrap the inspected item are

$$\eta(21) = \eta(11) = c_{s\_nc}\frac{P(LE < C < L)}{\beta} + c_{s\_c}\frac{P(C < \min(L, LE))}{\beta}$$

With regards to costs related to adjustment, at states (00), (10) or (20) the process is stopped for adjustment so

$$\phi(00) = \phi(10) = \phi(20) = c_a$$

As no adjustment are realized at states (01), (11) and (21) then $\phi(01) = \phi(11) = \phi(21) = 0$. Therefore, the average cost per item (in each cycle of inspection where ($m - 1$) items are sent to the customer or next stages of production) is given by:

$$C(m, L) = \frac{\sum_{w=0}^{2}\sum_{v=0}^{1} z_{(wv)} T_{(\mathrm{wv})}}{m-1} \tag{11}$$

The optimal values for m and L are obtained by numerical methods (search method) by minimizing (11), ie

$$\left(m^0, L^0\right) = \arg\min_{(m,L)} C \tag{12}$$

A Numerical Example

To illustrate the proposed model, consider a process production of T-shirts manufactured by a company. Large quantities of these items are produced and the quality control is evaluated by monitoring the number of non-conformities in the inspected piece. It is assumed that the difference of styles is negligible. The quality characteristic of interest follows a Poisson distribution, which parameter is the average frequency of non-conformities in the inspected piece (or item). In the inspection process the presence of stains and/or holes, ragged stitching, finishing problems, among others, on T-shirt are considered as defects that make bad the quality of the product.

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The parameters used are provided by the customer requirements (as historical data) or from the manufacturer. In this case, some parameters were set according to historical data in Table 1.

The manufacturer is increasingly concerned with reducing their costs, he wants to inspect the m-th T-shirt after every m produced items. He has interest in determining the size of the inspection interval (m^o) and the control limit (L^o) such as to minimize the average cost of production per item produced to detect changes in the average frequency of defects from $\lambda_0 = 2{,}5$ to $\lambda_1 = 6{,}5$. In this case, the client specified the upper specification limits $LE = 5$ to the manufacturer as shown in Table 1. In this scenario, the optimal parameters are got by the direct research and they are: optimal control limit L ($L^0 = 6$) and optimal sampling interval m ($m^0 = 88$) with a minimum average cost [C($) = 0.3004]. A comparative study of the average cost of the current proposal with other competing strategies can be made. For example, one possibility is if the manufacturer does not employ any control scheme, that is, all production is shipped to the customer without inspection. In this scenario, as we consider long run production terms, m^0 goes to infinity. Therefore, the probability of the process to produce non-conforming parts will increase, unless an adjustment is made. Numerically, we have: $1 - p_2 = P\,(C > 5 \mid \lambda = \lambda_1) = 0.6310$ although $\pi = 10^{-4}$. In this policy, the average cost per produced item is:

$$C^*\left(m,L\right)=\frac{1}{m}\left[m\cdot P(C>5)|\lambda=\lambda_1)\cdot c_{nc}\right]=3.1548$$

The other strategy is to calculate the average cost per item produced C($), without inspection, (all products are sent to the client without any type of verification), but the adjustment is performed at every 88 items produced (m^0). Similar to the previous case, the minimum average cost is equal to

$$C^{**}\left(m,L\right)=\frac{1}{88}\left[88\cdot P(C>5)|\lambda=\lambda_1)\cdot 5+100\right]=4.2911$$

In both strategies, average costs are greater than the presented proposal. Even inspecting and discarding costs of an item are both equal zero, the minimum average cost would decrease to $ 1.4738, considering that the process is adjusted at every 88 produced pieces. But if the cost of adjustment is also zero then the cost would decrease to $ 0.2513. But these cases are unfeasible in practice.

Table 1. Parameters according to historical data.

Costs	Values ($)	Process parameters	Values
c_i	0.025	π	0.0001
c_{nc}	5	λ_0	2.5
c_a	100	λ_1	6.5
C_{s_nc}	1	LE	5
C_{s_c}	2		

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Plots of costs versus m (L) varying L (m) are in Figure 2a, b. According to Figure 2a, lower costs are observed for moderate values of *L* (in this case *L = 5* and *6*) and m [*m* between 50 and 150]. In Figure 2b, the cost C($) decreases as L increases when L are in the range [0;8] approximately. (This range depends on the value of m). Also in Figure 2b, increase in L yields an increase in the cost C($) mainly if $L > LE = 5$ (according to Table 1). This result is also expected, since in this range of values of L, the frequency of sending of non-conforming shirts to customer increases. One should note that values of L larger than λ_1 are not desirable.

In the sensitivity analysis the average cost, the optimum sampling interval and the control limit are obtained varying one parameter at a time. Table 2 shows the results varying the values of λ_1. For values of $\lambda_1 > LE = 5$, L^0 increases but m^0 remains almost stable for $\lambda_1 > 10$. For $\lambda_1 \leq LE$, there is an increase in m^0 (when λ_1 decreases) indicating a lower frequency of inspection. And as expected, the minimum average cost decreases as λ_1 increases.

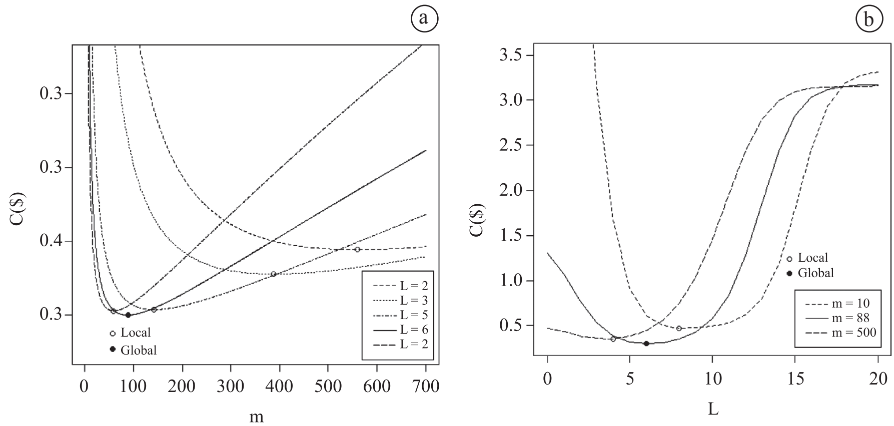

Figure 2. a) Plot of m × C($); b) Plot of *L* × C($).

Table 2. Sensitivity analysis: varying λ_1 keeping other parameters fixed.

λ_1	L^0	m^0	C($)	
3	3	896	0.2786	$\lambda_1 \leq$ LE
4	5	141	0.3097	
5	6	77	0.3119	
6.5	6	88	0.3004	$\lambda_1 >$ LE
8	6	99	0.2909	
10	7	87	0.2788	
11.5	7	94	0.2738	
15	8	93	0.2674	
30	12	94	0.2645	
45	15	94	0.2645	
90	23	94	0.2655	

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Table 3 shows the behavior of the average cost (minimum) per item produced when each of the parameters and costs of the process varies individually in a range of predefined values.

As shown in Table 3, variations in the parameters of the process and cost provoke significant changes mainly in the average cost and the sampling interval, but only slight changes in the control limit.

An increase in the inspection cost c_I, yields also an increase in the cost C($) due to the increase in m^0. This result is expected since as m increases, the frequency of inspection is reduced and then more time for a sign that the process may be operating out of control.

An increase in the cost of non-conforming c_{nc} results an increase in the cost C($) and an the reduction in m^0. (More inspections are performed more frequently).

As expected an increase in the cost of adjustment c_a, yields simultaneously an increase in the control limit L^0 (delaying an adjustment) and a reduction of sampling interval m^0 (more frequent inspections).

As lower is the cost to scrap a conforming item c_{s_c}, lower average cost and lower sampling interval (more frequent inspection) but with a large control limit L. When the cost of scrapping a non-conforming items c_{s_nc} increases, m^0 and C($) also increase and L^0 remains unchanged.

Table 3. Values of C ($), L^0 and m^0, varying one parameter at a time.

$\pi \times 10^{-3}$	C($)	L^0	m^0	λ_0	C($)	L^0	m^0	λ_1	C($)	L^0	m^0
0.01	*0.2366*	*6*	*271*	**1.5**	*0.090*	*5*	*87*	**4.0**	*0.3097*	*5*	*141*
0.1	*0.3004*	*6*	*88*	**2.5**	*0.3004*	*6*	*88*	**6.5**	*0.3004*	*6*	*88*
1	*0.5593*	*6*	*30*	**3.0**	*0.5236*	*7*	*77*	**11.5**	*0.2738*	*7*	*94*
10	*1.8628*	*6*	*15*	**4.5**	*1.6210*	*8*	*95*	**22.0**	*0.2648*	*10*	*94*
20	*2.7694*	*5*	*30*	**5.0**	*2.0581*	*8*	*136*	**34.0**	*0.2645*	*13*	*94*
c_I	**C($)**	**L^0**	**m^0**	**c_{nc}**	**C($)**	**L^0**	**m^0**	**c_a**	**C($)**	**L^0**	**m^0**
0.0025	*0.3001*	*6*	*87*	**0.5**	*0.0552*	*6*	*287*	**10**	*0.2654*	*4*	*119*
0.025	*0.3004*	*6*	*88*	**2.0**	*0.1442*	*6*	*139*	**100**	*0.3004*	*6*	*88*
0.25	*0.3030*	*6*	*90*	**5.0**	*0.3004*	*6*	*88*	**500**	*0.3706*	*7*	*78*
0.5	*0.3057*	*6*	*93*	**20.0**	*1.0131*	*6*	*44*	**1000**	*0.4358*	*8*	*54*
2.5	*0.3239*	*5*	*169*	**50.0**	*2.3707*	*6*	*28*	**5000**	*0.8785*	*9*	*49*
c_{s_c}	**C($)**	**L^0**	**m^0**	**c_{s_nc}**	**C($)**	**L^0**	**m^0**	**LE**	**C($)**	**L^0**	**m^0**
0	*0.2573*	*8*	*15*	**0**	*0.2993*	*6*	*87*	**3**	*1.3056*	*6*	*83*
2	*0.3004*	*6*	*88*	**1**	*0.3004*	*6*	*88*	**4**	*0.6397*	*6*	*82*
5	*0.3259*	*5*	*173*	**2**	*0.3014*	*6*	*89*	**5**	*0.3004*	*6*	*88*
10	*0.3504*	*5*	*214*	**5**	*0.3045*	*6*	*92*	**6**	*0.1516*	*6*	*99*
20	*0.3857*	*4*	*379*	**10**	*0.3094*	*6*	*96*	**7**	*0.0906*	*6*	*119*

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Moreover, higher the probability of a change of state π, the greater the tendency to operate the process under the state II, which justifies the increased cost C($).

To complete the first part of sensitivity analysis it would be interesting to identify among the parameters of the process (π, $\lambda_{0,}$ $\lambda_{1,}$ LE) and the costs (c_i, c_n, c_a, c_{s_nc}, c_{s_c}) which ones produce more impact in the average cost. In this sense a regression analysis is naively employed considering as output variable the average cost in Table 3 and as explanatory variables the earlier listed nine ones. To get rid of the influence of the different scales they are standardized. The coefficients of this regression are obtained by the usual least squared method and summarized in Table 4. The significant ones according to p-values are bolded in Table 4. The most important factors pointed out by the regression analysis are: the probability π of a shift in the parameter of Poisson distribution; cost to send non-conforming items to the customers c_{nc}; the in-control parameter of Poisson distribution λ_0; the specification limit *LE*; and the cost of adjustment c_a.

To assess the impact of errors in various types of cost in this type of on-line control planning, an additional sensitivity analysis was performed. Minimum average cost and optimal parameters L^0 and m^0 were obtained considering a range of ±15% for each type of cost as shown in Table 5.

Of the total of 243 combinations, the value of L^0 was unchanged in all combinations. About the optimal interval, variations of the inspection intervals obtained with errors in the costs in relation to the real optimal value (which is $m^0 = 88$) were calculated. The maximum, minimum and average changes are, respectively, equal to 15.9%, 0% and 0.0004%. Similarly, the variation in the average cost obtained with errors in the various cost components around the minimum average cost ($ 0.3004) was also calculated. The maximum, minimum and average changes are, respectively, equal to 15%, 0.001% and 0.11%. These results reinforce a sense that the planning to control for number of nonconformities is robust to the variations in cost around 15%.

Table 4. Estimates of the coefficient in a naïve regression analysis.

Costs	Coefficient	p-values	Process parameters	Coefficient	p-values
c_i	–0.010	0.677	π	0.458	$<10^{-3}$
c_{nc}	0.349	$<10^{-3}$	λ_0	0.341	$<10^{-3}$
c_a	0.084	0.002	λ_1	–0.025	0.321
C_{s_nc}	–0.013	0.591	LE	–0.153	$<10^{-3}$
C_{s_c}	0.002	0.944			

Table 5. Cost values used in the complementary sensitivity analysis.

	c_i	c_{nc}	c_a	c_{s_c}	c_{s_nc}
–15%	0.02125	4.25	85	1.7	1.7
Values in Table 1	0.02500	5.00	100	2.0	2.0
+15%	0.02875	5.75	115	2.3	2.3

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Conclusions

In this paper an economic model is developed to monitor the rate of non-conformities. The parameters: the interval sampling m^0 and upper control limit L^0 are determined that minimize the average cost per item produced. It is assumed the quality of characteristic of interest, the number of nonconformities follows a Poisson distribution and a long run production. Like Trindade *et al*. (2007a) and other earlier mentioned papers, one inspection is performed after a production of m items and that only the m-th is inspected and after discarded. If number of non-conformities of the inspected item is greater than L^0, the process is declared out of control and the production is stopped for adjustment otherwise the production goes on. From the sensitivity analysis the most important factors, which play important role on the average cost are: the probability π of a shift in the parameter λ_0, the specification limit LE and the parameter λ_0. The contribution of these analyses was of great importance, since allowed us to understand the process and subsequently the influence on the final cost of each parameter involved. Furthermore, we show that the strategy adopted here ensures a good product quality as a lower average cost of manufacturing these parts to the manufacturer.

It is important to point out that all the earlier mentioned papers: Quinino and Suyama (2002), Quinino and Ho (2004) and Trindade *et al*., (2007a, b); Quinino and Ho (2004) and Quinino *et al*. (2010) can be viewed as particular case of the proposed model if the optimized upper control is set equal to upper specification limit and also only a single inspection (r = 1) is performed instead of r > 1 repeated classifications as suggested in these papers to decrease the impact of error classifications.

Some suggestions for possible extensions may be suggested. One possibility is to adapt the proposed model in a case a short run production. In this scenario, the manufacturer has also interest in choosing m^0 and L^0 that minimizes the expected cost per item produced in a finite production of *n* items, however in that case some stationary results from Markov chain cannot be applied.

Another possible extension would be a model varying inspection intervals: a longer one k if the number of non-conformities is less than a discriminate limit D (but closer to λ_0) and a shorter interval m if the C is lower than but closer to L, for example. However, the problem becomes more complex with more parameters to be optimized: optimal limits L and D and two inspection intervals (optimal k and optimal m) such that minimize the average cost of items produced, with the restriction k > m. After one adjustment, the process returns to make the first inspection after a production of k items and the following interval inspection depends on the results of the previous inspected item.

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