Wellbore Stability Analysis Based on Statistic Analysis Applied to a Probabilistic Method

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ABSTRACT: When developing a wellbore stability analysis project, it is important to have as much information about the area to be drilled as possible due to the importance of data accuracy to construct the mud weight window. Usually, in order to avoid drilling events, data of adjacent wellbores are used to deterministically estimate the fracture and collapse gradients and establish a mud weight range that assures the wellbore stability along the desired trajectory. This paper presents a wellbore stability analysis based on a probabilistic method - First Order Second Moment (FOSM) - that considers a statistic analysis to characterize the uncertainties associated to the failure pressures calculated. First, a deterministic method is used to calculate the fracture and collapse gradients. The input parameters are assumed to have a standard deviation of 10% of the mean value. Then, the results of the first step are used as input data to the probabilistic method that will provide scenarios for different probabilities of success (P10, P50 and P90). The probabilistic method proposed is based on a statistical analysis to define the standard deviation associated to the gradients’ calculus. This paper will approach two different ways to define this standard deviation: normal variation and random variation. Once the analysis of the results is made, it is possible to conclude for the probabilistic method which scenario presents a more conservative behavior, the standard deviation of the results and it will be possible to compare the two statistical analysis described for each of the failure model.

KEYWORDS: Stability, Gradient, Fracture, Collapse, Statistical analysis

1 Introduction

Thousands of wellbore drilling operations take place every year around the world. At some point, to make it possible, a study of the fracture, upper collapse and lower collapse pressures, and also the geopressures shown in the field was made with the purpose of developing a wellbore stability analysis project, which is an important phase when planning the drilling operation. One of the first steps when elaborating the stability project is collecting data from adjacent wellbores and make a correlation, based on lithology, from these data and the wellbore to be drilled. Since the data used in the wellbore project to calculate the fractures and collapses gradients are from other wellbores in the field and are indirect measures, they carry uncertainties that can lead to errors in the operational window, bringing on the occurrence of drilling events. In order to avoid drilling events and their consequences – such as non-productive time –, probabilistic and statistic methods can be used to minimize the error propagation initiated in the input parameters.

This paper aims to compare two different statistical approaches to define the uncertainties associated to the input parameters, and observe and compare their influence in the final outcome - fracture and collapses.
gradients - when applied in a probabilistic method. First, the fracture and collapse gradients will be calculated deterministically using the mean values of the input parameters, then, these results will be applied in a probabilistic method which will be supported by statistical analysis information. This statistical analysis will consider two types of distribution pattern for the same data basis - normal and random distribution. The range of variation of the input parameters obtained from each distribution will be used to calculate the standard deviation associated to the function of the gradients’ calculus. With the probabilistic methods, it will be possible to compare the results obtained when using the two different standard deviation to calculate the gradients related to the probability of failure of 10%, 50% and 90% - P(10), P(50) and P(90), respectively.

2 Methodology

In this paper, a probabilistic method will be used to compare the results for fracture and collapse gradient obtained when using different statistical analysis approaches to determine the uncertainties associated to the input parameters. Fontoura et al. (2002) described three probabilistic methods based on derivatives approximations and reliability indexes that enables the estimative of the mud weight window for different probabilities of failure. Since this paper aims to analyze the different statistical approach for the same problem rather than the probabilistic analysis, one of the probabilistic methods is used in this study: First Order Second Moment method, which is described in the following topic 2.1.

2.1 First Order Second Moment (FOSM)

This probabilistic method has been proposed, discussed and used for geotechnical problems by other authors before (Christian, 1994; Harr, 1987). The FOSM method is based on the safety factor and the reliability index \( \beta \). First the safety factor can be determined as it shows in equation (1) for fracture and upper collapse gradients and equation (2) for lower collapse.

\[
SF = \frac{P}{p_w} \quad (1)
\]

\[
SF = \frac{p_w}{P} \quad (2)
\]

Where SF is the safety factor, P is the fracture or collapse gradient and \( p_w \) is the inner wellbore pressure. Then the reliability index (\( \beta \)) is described as a function of the safety factor, as it shows in equation (3).

\[
\beta = \frac{\bar{SF} - 1}{\sigma(SF)} \quad (3)
\]

Where \( \bar{SF} \) is the mean value of the safety factor, obtained with mean values of the input parameters and \( \sigma(SF) \) is the standard deviation that can be obtained as it shows in equation (4).

\[
\sigma(FS)^2 = \sum_{i=1}^{n} \left( \frac{\partial SF}{\partial X_i} \right)^2 * \sigma(X_i)^2 \quad (4)
\]

Where \( X_i \), with \( i = 1: n \), are the input parameters and \( \sigma(X_i) \) are the standard deviation associated with each input parameter. The derivatives required in equation (4) can be approximated as it shows in equation (5).

\[
\left| \frac{\partial SF}{\partial X_i} \right| = \frac{|SF_i - \bar{SF}| + |SF_i - \bar{SF}|}{2*\delta X_i} \quad (5)
\]
Where $SF_i^+$ is the safety factor obtained with a positive variation of the input parameter, $SF_i^-$ is the safety factor obtained with negative variation of the input parameter and $\delta X_i$ is the imposed variation in the input parameters that caused the outcomes of $SF_i^+$ and $SF_i^-$. After the definition of the reliability index $\beta$ and considering a normal distribution for the possible scenarios obtained, one can calculate the fracture and collapses gradients for different probabilities of failure using normal cumulative distribution function (Pinheiro, 2012; Spiegel, 1978) as it shows in equation (6), where PR is the probability of failure. The normal cumulative distribution function ($\phi(x)$) is described as a function of the error function (erf) as it shows in equation (7).

$$PR = 1 - \phi(\beta)$$

$$\phi(x) = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{x}{\sqrt{2}} \right) \right]$$

2.3 Statistical Analysis

As shown in the formulations presented in equations (5), it is required to set a variation for the input parameters, so it is possible to calculate the approximated derivatives. To define this variation, it is necessary to fulfill a statistic analysis of each input data, hence recognize what kind of distribution these datasets follow. This previous statistical analysis to identify the pattern behavior of the uncertainties is an important step when applying a probabilistic method to not produce an incorrect mud weight window leading to a mistaken wellbore stability analysis, since the outcomes obtained from each kind of distribution are different.

To determine the which distribution best fit the approached scenario, one must collect as much information as possible from correlate wellbores and from the field and then, verify the histograms of frequency. In this paper, it will be presented two different distribution patterns to determine the variation that will be used in the probabilistic methods. These statistical analyses are described in the following topics 2.3.1 and 2.3.2.

2.3.1 Normal Variation

In this type of variation, it is considered a normal distribution for the input parameters. A normal distribution is characterized as a symmetric distribution where the mean, mode and median values match. Figure 1 shows an example of a normal distribution gamma ray data along the wellbore trajectory.

![Gamma Ray (API)](image)

Figure 1. Histogram representing a normal distribution of frequency.
In this study, it will be assumed that each parameter is characterized by a standard deviation of 10% of the mean value. In the FOSM method it will be imposed a variation of 5% of the standard deviation of each parameter. A positive variation will provide the values of $SF_i^+$ and negative variation will provide the values of $SF_i^-$.  

2.3.2 Random Variation  

This type of variation considers a random value for positive and negative increment within a specified range. A random distribution occurs when the data that is been analyzed do not follow a pattern. In a histogram, it is represented by bars that increase and decrease without any criterion. Figure 2 represents an example of random distribution of shear transit time data along the wellbore trajectory.

![Figure 2. Histogram representing a random distribution of frequency.](image)

For this study, input parameters are considered to have a standard deviation of 10% of the mean value. For FOSM method, it is chosen a positive variation randomly within the interval limited by $\bar{X}_i$ and $\bar{X}_i + 5\% \ast \sigma(\bar{X}_i)$, and it is chosen a negative variation randomly within the interval limited by $\bar{X}_i - 5\% \ast \sigma(\bar{X}_i)$ and $\bar{X}_i$.

3 Results and Discussion  

To perform this study, original data from a vertical wellbore were used as input for the probabilistic method and the outcomes for both kind of variations described were analyzed. Figure 3 Figure 5 shows the results for fracture, upper collapse and lower collapse gradients respectively. It was used Kirsch’s formulations and Mohr-Coulomb criteria to implement this research. Since the methodology assumes a normal distribution for the possible results for fracture, upper collapse and lower collapse gradients, the curves assigned as 50% of failure probability (black in fracture graphic, purple in upper collapse graphic and green in lower collapse graphic) for both of variation types, match with each other and also with the results obtained with mean values of the parameters.

A Matlab code was developed to implement the probabilistic method with each statistical analysis. The parameters used as input data for this code were minimum horizontal stress azimuth, pore pressure, maximum horizontal stress, minimum horizontal stress, vertical stress, internal friction angle, tensile strength, unconfined compressive strength, Biot’s coefficient and Poisson’s ratio. Each failure model code was developed separately for each statistical distribution approach, thus, it was possible to compare the computational performance for each scenario. The time required to perform a specific failure model did not have a significant difference between the two statistical analysis. The difference only appears when changing the failure model; the fracture
Gradients took less than a minute to be calculated (regardless it was normal or random distribution), on the other hand, to calculate the collapses – upper and lower – it took about 1.2 min. This fact is due the higher complexity of the collapses calculus.

Figure 3 presents the results obtained from FOSM method. As expected, it can be noticed that independently of the variation type selected for the input parameters, the curves that assure a 10% of failure probability are more conservative than the ones with 90%. When comparing the two variation types for different failure probabilities, the results shows that for P(10), normal variation are more conservative than random variation. However, this scenario changes when the failure probability is 90%, which can be explained by the difference in their standard deviation, since the mean standard deviation for normal variation results is 4.18 ppg and for random variation is 1.95 ppg.

![Fracture gradients obtained from FOSM method.](image)

**Figure 3.** Fracture gradients obtained from FOSM method. The curves in red are the results for Normal variation of the parameters and the curves in blue are the results for random variation of the parameters.

Figure 4 shows the outcomes for FOSM method to calculate the upper collapse gradient. From this results, it is possible to verify that the response for the upper collapse produced by FOSM method are very similar for both type of variation described in this paper, once the standard deviation for normal and random variation are 3.78 ppg and 4.11 ppg, respectively.
Figure 4. Upper Collapse gradients obtained from FOSM method. The curves in red are the results for Normal variation of the parameters and the curves in blue are the results for random variation of the parameters.

From Figure 5, it can be observed that the results for lower collapse do not present significant difference for normal variation or random variation. When comparing these results from the other failure model, it is possible to affirm that this scenario presents a lower standard deviation (1.42 ppg for normal variation and 1.81 ppg for random variation).

Considering the results presented, it is possible to conclude that for fracture calculus, the kind of distribution chosen for the input parameters makes a significant difference and, even considering the results for collapses similar, it is important to emphasize the relevance of a previous statistical analysis of each parameter when using a probabilistic method to estimate the mud weight window. A bad decision about the parameters that lead to the operational window may result in problems during drilling operation, and, consequently in non-productive time. Many extra expenses can be avoided if the input parameters are well treated. Since this research aims to show how different can be the results of fracture and collapses gradients for different statistical analysis, it was presented extreme scenarios: one with all data characterized by normal distribution and other with all data characterized by random distribution. However, when analyzing each particular data, it is possible to verify different distributions in the same scenario, hence, the statistical methods can be combined to produce a mud weight window precise and coherent.
4 Conclusions

Considering the uncertainties associated to the input parameters when calculating the operational window is important once the results presented by the use of mean values can consider safe a range of mud weight window that, in reality, is not. In this paper, a probabilistic method based on statistical analysis of the input data and their uncertainties were applied and two paths were introduced: normal and random distribution of the parameters.

By analyzing the results presented, it can be concluded that:

- In this case study, for upper and lower collapses, the results obtained by using standard deviation from normal variation or random variation of the input parameters are very similar when using the probabilistic method proposed. However, the standard deviations of the results are significantly higher for the upper collapse.
- For fracture gradient, the outcomes proved to be different for the two statistical approaches proposed, since for normal variation, the results presented a 4.18 ppg for mean standard deviation against 1.95 ppg
presented by the random variation approach.

- Although $P(10)$ offers a safer scenario, it may not be feasible in practice, since the results for this probability of failure proved to be too conservative, hence, providing a very small operational window.
- The computational performance required to execute both methods are practically the same.
- When using a probabilistic method it is important to analyze each input parameter in particular to define which distribution best fits that parameter, and by doing this it is possible to avoid drilling events, hence, extra costs during drilling operation.

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